

Teacher Perspectives on Mathematics Content and Pedagogy: Describing and Documenting Movement

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Abstract

This paper reports on a study examining teacher perspectives on mathematics content and pedagogy after one year of teacher participation in a mathematics professional development program. Teacher narrative responses to questions about student work samples provided qualitative data for analysis, where teachers provided input on student thinking and instructional decisions. Responses revealed teacher movement in the areas of the professional development program's goals: a) movement from a strict and superficial procedural mathematics perspective toward a richly connected and integrated procedural/conceptual perspective; and b) from a teacher-directed pedagogical perspective toward a learner-responsive pedagogy. Two additional findings included: a) content-specific issues in teachers' response; and b) a recognition of the utility and usefulness of the coding analysis tool for documenting and describing teacher perspectives and movement, able to capture fluid movement on the continua of content and pedagogy perspectives.

Key words: Teacher Education, Content Perspectives, Pedagogy Perspectives, Pedagogy, Professional Development, Mathematics Education, Learner Responsive

1. Purpose of the Study

We know that teachers bring to their classroom practice many factors that influence their pedagogy. They bring knowledge, skills, and understandings of mathematics that impact student learning (Adler and Davis 2006; Ball, Lubienski, and Mewborn 2001; Fennema and Franke 1992; Hill and Ball 2004; Hill, Rowan, and D. Ball 2005). They also bring into their classrooms perspectives on the nature of mathematics and how it should be taught.

Perspectives on mathematics include whether it is a procedural or a conceptual activity, whether it is necessary to know mathematics both conceptually and procedurally, and whether there is some combined way to know mathematics (Baroody, Feil, and Johnson 2007; Star 2005). Perspectives on mathematics pedagogy include whether it is better taught with a teacher-directed or more learner-responsive approach, or if one can use and apply both approaches. In this paper we report on a study exploring teacher change in terms of perspectives on mathematics and pedagogy with teacher participation in a professional development project. We examine these particular perspectives because the program serving as the context for this work has among its goals to support teacher movement along a continuum from a strict and superficial procedural perspective on content to a more richly connected and integrated procedural/conceptual perspective; and along the pedagogy continuum from a teacher-directed to a learner responsive perspective.

In the following paragraphs, we define the perspectives on mathematics and pedagogy that are central to this work, and review the literature on which those definitions are based. We then describe the methods utilized in our study, including the context of the work, the nature of the data, and data analysis procedures. Finally, we discuss findings and closing thoughts.

2. Theoretical Framework

Supporting teachers' practice toward students' mathematics learning necessitates consideration of multiple concepts brought to the teaching of mathematics. The area of teacher mathematics content knowledge already has a deep base in the literature. What we bring to the discussion about teachers' perspectives on mathematics content and pedagogy is a distinction between perspectives and what is generally understood in the literature as 'disposition'.

Scholars define dispositions as traits that lead a person to follow certain choices or experiences (Damon 2005) or as tendencies to exhibit frequently a pattern of behavior directed to a broad goal (Katz 1993). For Gresalfi and Cobb (2006), the word "disposition" encompasses ideas about, valuing of, and ways of participating with a discipline. It is about identifying with a discipline and how it is realized in the classroom. A dictionary definition of disposition describes it as "a person's inherent qualities of mind and character" (New Oxford American Dictionary 2005). Similarly, a perspective is defined as "a particular attitude toward or way of regarding something; a point of view; the state of one's ideas" (Oxford Dictionaries 2010). Perspective, although sometimes related to attitudes and beliefs in the literature, is usually recognized as being a result of experience, as something that can change, and something that influences practice. (e.g. Ross 1986; Ross and Smith 1992; Zeichner and Tabachnick 1983).

In our work we chose not to name the mathematics and pedagogy concepts we study dispositions because definitions of dispositions as cited above suggest less the likelihood that they can change than does the meaning of perspective. As educators, we believe that the perspectives teachers bring to bear on mathematics and pedagogy are not "inherent" but have been learned through lived experiences in and out of school. Additionally, our work in professional development relies in part on the belief that professional development can result in teacher change in terms of perspectives on content and pedagogy.

2.1 Mathematics Content and Pedagogy Perspectives

Contrary to what many in the general public believe, the content of mathematics is much richer than only arithmetic or computation; and learning mathematics content with understanding is a more complex endeavor than merely knowing the "how to do" of mathematics (Heibert et al. 1997). These and other perspectives are on the opposing "sides" of what Jon Star (2005) calls the "so-called math wars" (p. 404), about which he cites Judith Sowder's statement that "Whether developing skills with symbols leads to conceptual understanding, or whether the presence of basic understanding should precede symbolic representation and skill practice, is one of the basic disagreements" (1998, as cited in Star 2005).

In the literature we find research suggesting that the sequence in which students experience the procedural and conceptual components of mathematics impacts student learning (Pesek and Kirschner 2007). However, Pesek and Kirschner's work does not suggest a prioritization of procedural or conceptual approaches to learning mathematics, only something about the relationship between the two in a learning context. Star (2005) advocates for procedural understanding, but does so from the position that both procedural and conceptual understandings are viewed too simplistically: Conceptual understanding as rich and concrete and procedural as superficial and lacking connections. Star suggests a framework where both knowledges are studied for their rich, connected, and deep relational and integrated qualities.

Baroody, Feil, and Johnson (2007) suggest a conceptualization that is consistent with Star's "recommendation to define knowledge type independently of the degree of connectedness" (p. 123). Baroody et al. propose the following definitions of procedural and conceptual knowledge: a) Procedural knowledge refers to "mental 'actions or manipulations', including rules, strategies, and algorithms, for completing a task" (de Jong and Ferguson-Hessler, p. 107 as cited in Baroody et al., p. 123); b) Conceptual knowledge is "knowledge about facts, [generalizations], and principles" (de Jong and Ferguson-Hessler, p. 107 as cited in Baroody et al., p. 123).

Baroody et al. (2007) distinguish their conceptualization with degrees of depth/superficiality, connectedness, and mutual dependence/independence and note: "depth of understanding entails both the degree to which procedural and conceptual knowledge are interconnected and the extent to which that knowledge is otherwise complete, well structured, abstract, and accurate" (p. 123). We take our content perspectives from this literature, and assign a range of conceptualizations, from procedural to integrated procedural/conceptual, to form a continuum of perspectives on mathematics content for our study.

As already noted, our work also includes investigations on perspectives on mathematics pedagogy, grounded in ways similar to that of perspectives on mathematics content. The NCTM, particularly by way of its standards publication (2000), puts forth a vision of school mathematics "where all have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all ... Knowledgeable teachers have adequate resources ... The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding" (NCTM 2000, p. 3). This vision, which includes learning mathematics with understanding (Heibert et al. 1997), has been embraced by much of the mathematics education community. It is in direct contrast to what one might describe as traditional teacher-directed mathematics instruction that often includes less visible student engagement in the learning process. That is not to say that students are not cognitively engaged when in a teacher-directed learning environment; but it does mean that students in teacher-directed environment are not communicating mathematical ideas, not drawing on reflective practices and deep understanding. Instead, the teacher directs students on what to do, on what and how to learn, playing the dominant role; and students respond to the teacher by following instructions (Eccles, Midgley, and Alder 1984; Gmitrová and Gmitrov 2003).

Also on the mathematics pedagogy continuum is student-centered instruction which is "[d]esigned to elicit and build on students' ways of understanding mathematics" (Empson and Junk 2004, p. 122) and is often problem-based (Ma and Zhou 2000). Student-centeredness includes the teacher talking less and the learner talking more, with the learner doing the mathematical thinking and having opportunities to self-correct and generate knowledge through rich mathematical practices. Student-centered instruction is further emphasized in the latest Common Core State Standards Initiative (2010) for mathematics wherein the teacher seeks students' understanding of mathematics, not merely agility with procedures, by listening to student justification of solutions.

The context of our research was a mathematics education professional development project that had a goal to move teachers beyond teacher-directed and even student-centered instruction to what we call learner-responsive pedagogy (LRP). As in student-centered pedagogy, the LRP teacher makes decisions based on the learner's interests and focused on the learner's active engagement in the lesson. Teaching mathematics for understanding requires teachers to assess student understanding by inviting their justification of solutions in order to ascertain their understanding and make future instructional decisions based on these explanations (Common Core State Standards Initiative 2010). So LRP includes two additional distinctive qualities that separate it from problem/activity-based or student-centered approaches: a) a shift, or expansion of each constituents' responsibilities and roles in the learning process from teacher as authority to authority shared by teacher and learner (Freire 1973/1989); and b) action: a resulting and deliberate instruction based on teacher knowledge of learner thinking and understanding.

With on-going analysis of student thinking as a fundamental component of LRP, this pedagogy requires a kind of engagement by the students and the teacher where students can provide evidence of the nature and depth the teacher needs in order to base instruction on that evidence. This engagement necessitates a learning environment that is open and safe enough for the students to be comfortable expressing their understandings as well as their challenges. The environment is one where the pursuit of inquiry and reflection by both the teacher and the students is understood to be fundamental. In this environment, students find activities providing them with ample opportunities to engage with the discipline. And this environment is conducive to teachers making valid judgments regarding student learning, based on rich evidence. This means that the environment includes teachers with rigorous knowledge of the content of mathematics, to support teacher recognition of student knowledge and lack thereof. This is an environment where instructional decisions are made in direct response to the learners' understanding; and pedagogical action, based on the learner's needs, is intentional. For this necessary exchange of information, the teacher and student must be in the kind of relationship where they are not just safe, comfortable, and knowledgeable; they must also be able to share authority, where, as Freire (1973/1989) describes it, the teacher becomes the students and the student becomes the teacher.

These pedagogy perspectives of teacher-directed, student-centered, and learner-responsive form the continuum of pedagogical perspectives in our study.

3. Methodology

3.1 Participants, Sampling, Context, and Data Source

Participants in this research project are teachers in schools enrolled in a mathematics education professional development program over one academic year. The professional development program provided teachers opportunities to work with a mathematics coach assigned to the teachers' schools. All research participants are certified or licensed teachers, and are credentialed to teach in any of grades one through eight; some also are credentialed to teach kindergarten. They teach in elementary, intermediate, or middle schools, and were free to choose whether or not to participate in the research, regardless of their participation in the professional development program.

In the academic year of the teachers' professional development and this research project, 143 teachers consented to participate in the research. Of the 143 consenting teachers, 100 responded to student work items in the autumn and again in the spring of the academic year, with pre-professional development and post-professional development responses. From the set of 100 participants with both pre- and post-professional development responses, we created a purposeful, random sample (Patton 2002) of 20 participants for analysis. We based our sampling strategy on the following principles, where we:

- a) Included all responders with narrative responses on all items making the sample purposeful in its inclusion of only full sets of responses (Patton 1990).

- b) Randomized within the purposeful sample to assure a representative set.
- c) Limited participation to 20 participants: 20 participants, each with 20 response in each of two administrations generated 800 data points for analysis, a significant number of data points for qualitative analysis, given the practicalities of time constraints, collaborative coding, and inter-rater reliability goals of the work (Patton 2002).

The professional development program's goals included to shift teacher perspectives: a) away from a strict (and superficial (Star 2005)) procedural perspectives on mathematics and toward richly connected procedural/conceptual perspectives; and b) away from teacher-directed perspectives on pedagogy and toward a learner-responsive pedagogy perspective.

The data source used for our analysis is a questionnaire generating teacher analyses of student work. Participants provided narrative responses to two questions for each of ten student work samples. One question asked for teacher interpretation of the student's thinking and the second question asked for teacher suggestion of next instructional moves. For example, one item provides the following student work:

Bobby was given the problem $17 - 9 = \underline{\quad}$ and solved it as follows:
 $17 - 9 = 17 - 10 - 1.$

Teachers were asked to a) provide a rationale to describe what Bobby was thinking; and b) provide an explanation of what to say or do to help Bobby further his thinking.

For each student work sample, questions posed for teacher analysis were designed to be specific to the particular sample of student work, but always with the first question focusing on student thinking and the second on pedagogical decisions.

3.2 Data Analysis

We conducted coding analyses on participants' extended responses, through each of the two lenses of mathematics content perspectives and mathematics pedagogy perspectives on both the autumn (pre) and spring reviews (post) responses. We also conducted a comparison analysis of the pre and post responses. We coded responses on student thinking for mathematics content perspectives on the Procedural to Conceptual to Integrated Procedural/ Conceptual perspectives continuum. We coded responses about instructional decisions from Teacher-Directed to Problem-Based or Student-Centered to Learner-Responsive Pedagogy on the mathematics pedagogy perspectives continuum. The reader may see the abbreviated codebook of Table 1 for the list of codes. Each student thinking response had one or more content codes assigned to it; and each instruction response had one or more pedagogy codes assigned to it. Multiple researcher reviews of teacher responses, collaboration on coding assignments, and comparisons to coding by an outside reviewer (Morse, Barrett, Mayan, Olson, and Spiers 2002) helped reach inter-rater reliability goals of 85% in the analysis.

Code	Mathematics Content Perspectives
IPC	Integrated procedural/conceptual perspective: “to view meaningful knowledge of mathematical procedures and concepts as intrinsically and necessarily interrelated, not as distinct categories of mathematics” (p. 127 Baroody, Feil, and Johnson 2007).
C	Conceptual perspective: Jong and Ferguson-Hessler’s 1996 definition is that this is: knowledge about facts [generalizations], and principles” (p. 107), cited in Baroody et al. 2007; and how these are related or interconnected.
P	Procedural perspective: Jong and Ferguson-Hessler’s 1996 definition is that this consists of “mental actions or manipulations” (p. 107) including rules, strategies and algorithms for completing a task” (Baroody et al. 2007).
IO	Incorrect/other: incorrect understanding of the concept or knowledge of procedures; or does not clearly belong in any of the other categories.
Code	Mathematics Pedagogy Perspectives
LRP	Learner-Responsive Pedagogy: Based on student learning (assessment). Must include focus on interrogating student understanding and basing instructional decisions on student cognition. It necessarily requires more ownership and contribution on the part of the learner and less control and authority on the part of the teacher. Includes elements of liberatory pedagogy (Freire 1973/1989).
PSC	Problem/activity-based and Student-Centered instruction is “[d]esigned to elicit and build on students’ ways of understanding mathematics” (Empson and Junk 2004, p. 122) and is often problem-based (Ma and Zhou 2000).
TD	Teacher Directed: Teacher directs students on what to do, what and how to learn. The teacher is dominant role; and the students respond to the teacher by following instructions; what Freire (1973/1989) defines as “banking” education.
O	Other does not clearly belong to any of the other categories, typically because the response was tangential to the topic, the meaning wasn’t clear in terms of the pedagogical approach, or the response missed the point of the problem.

Table 1. Abbreviated Codebook

4. Results

4.1 Movement in Content and Pedagogy Perspectives

Table 2 includes results of the coding analysis of participant perspectives on mathematics content, showing movement or lack thereof on a Procedural (P) to Conceptual (C) to Integrated Procedural Conceptual (IPC) continuum. Data points are best viewed as clustered around or tending toward a particular position, allowing for some variance in the content and pedagogy perspectives, while still describing a location.

Percent of Positive Movement		Percent Negative Movement		Percent of No Movement	
P to C	5%	IPC to P	15%	Remain P	20%
P to IPC	25%			Remain IPC	10%
				Remain P & IPC	25%

Table 2. Participant Movement on Mathematics Content Continuum

As the data in Table 2 shows, 25% of the participants tended to exhibit positive movement from Procedural (P) to Integrated Procedural Conceptual (IPC) perspectives on mathematics content. Consider a student work sample:

Jenny uses the following method to find 28% of 60,000 mentally:
 20% is $\frac{1}{5}$ and $\frac{1}{5}$ of 60 is 12, so 20% of 60,000 is 12,000.
 One percent of 60,000 is 600, and that times 8 is 4800.
 So the answer is $12,000 + 4,800$, which is 16,800.

We coded the following teacher’s response to this sample regarding student thinking as Procedural (P): “When finding answers to problems mentally it is easier to break it down into easier chunks.” This response is procedural because it only notes what one would do. Later in the year, that same teacher responded to the same item with, “Jenny broke apart the problem into easier chunks. She understands the relationships between percents and fractions and understands that you can divide 60 by 5 to show $\frac{1}{5}$.” We coded this response Integrated Procedural Conceptual (IPC) because it included both procedural and conceptual components and because the conceptual and procedural components are connected.

Another item on the questionnaire focused on the student work sample on the Toothpick Problem (See Figure 1).

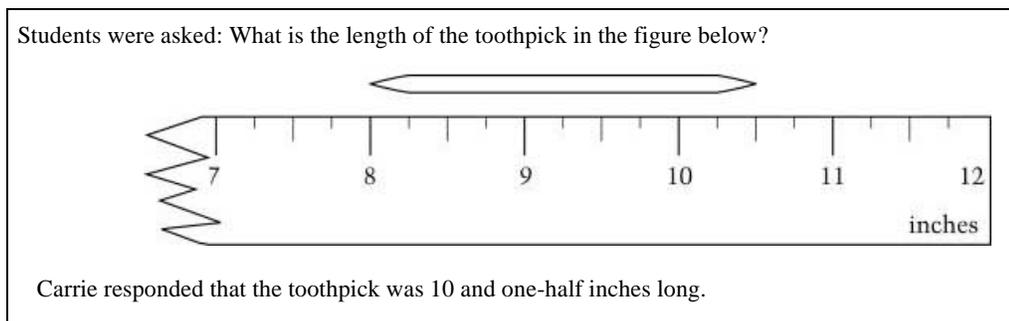


Figure 1. Carrie’s Toothpick Problem Response

We coded the following response to the question of what the student was thinking: “I think she doesn't understand that you start at the beginning of the ruler to measure” as a Procedural (P) response; but coded another response of “She seems to be able to read the ruler but is struggling to understand how to measure when an object doesn’t begin at 0” as an Integrated Procedural Conceptual (IPC) response because of the teacher’s analysis that notes the concept of the starting point of a measure.

On a division of fraction problem (Shanna’s Problem), the student in the work sample used a mathematically valid alternative algorithm to find a correct answer. See Figure 2.

Shanna's solution to the problem $\frac{3}{4} \div \frac{1}{8} = ?$ was the following:

Shanna: I think the answer is $\frac{12}{16} \div \frac{2}{16} = \frac{6}{1}$ and my method will always work.

Figure 2. Shanna’s Fraction Problem Solution

For this item, an example of negative movement for a teacher research participant is found in the following pre- and post-professional development responses:

Pre-professional development teacher response: “It seems she made common denominators.” We coded this response as a Conceptual (C), because it simply mentioned a conceptual component. It did not suggest a procedure, and thus could not be Procedural (P) or how some procedure might be connected to the concept (and thus would not be Integrated Procedural Conceptual (IPC)).

Post professional development, that same teacher responded with “Common denominators are used for adding or subtracting fractions. She doesn’t understand that you divide fractions by multiplying the reciprocal of second fraction,” which we coded as Procedural (P) because it focused on the procedure, without explaining meaning.

Table 3 includes the proportional results of the coding analysis of the qualitative data on participant perspectives on mathematics pedagogy. These results show percentages of movement on the Teacher-Directed (TD) to Problem/Activity-Based or Student-Centered (PSC) to Learner-Responsive Pedagogy (LRP) mathematics pedagogy continuum. In Table 3, 25% of the participants revealed positive movement from Teacher-Directed (TD) to Problem/Activity-Based or Student-Centered (PSC), and Problem/Activity-Based or Student-Centered (PSC) to Learner-Responsive Pedagogy (LRP). An example of such positive movement from the problem about Bobby’s solution to the $17-9= \underline{\quad}$ problem cited previously starts with “By using his own explanation I could: 1. verify his mistake as I see it and 2. allow him to discover his own error and then he could recognize his error in the future” which we coded Problem/Activity-Based or Student-Centered (PSC) because it focuses on helping the student realize his error. Later in the year that same teacher’s response becomes “I would have him solve both sides using pictures or models and compare his answers. Using this method Bobby could see that his process is wrong. He can visualize the need to add that 1 back in the equation.” This end of the year instructional suggestion is coded Learner-Responsive Pedagogy (LRP) because the teachers knows and helps the student discover his errors by having the student compare and reflect upon his own work. The teacher and the student share the authority in the experience.

Percent of Positive Movement		Percent Negative Movement		Percent of No Movement	
TD to PSC	20%	LRP to PSC	10%	PSC	40%
PSC to LRP	5%	More TD	5%	TD	15%
				LRP	5%

Table 3. Participant Movement on Mathematics Pedagogy Continuum

An example of the 15% of the participants who showed negative pedagogical movement is as follows: A teacher’s first response about Bobby’s problem was, “Bobby I like how you rounded the 9 to 10. Now we have to subtract $17-10=7$ and add 1 back to get to the 9. Let me show you with our cubes what I would do.” We coded this response Teacher-Directed (TD), but with expectation of movement at the post administration because of the potential in the use of manipulatives. However, at the post administration, the same teacher responded, “If Bobby explains his answer to me then I would correct him when he explains -1 and tell him he needs to +1 because the problem was $17-9$ and show him with base 10 blocks - 9 cubes is less than 10 cubes or 1 long (10 cubes).” Bobby’s use of manipulatives was clearly still from a teacher-directed perspective, and perhaps even more Teacher-Directed (TD) in the language of “I would correct him,” “tell him,” and “show him.”

4.2 Emergent Findings from Analysis by Mathematics Content Strands

An additional review of the data by mathematics content strand revealed interesting results for two different NCTM content standards: number and operations; and data analysis and probability. Two of the items drawn from the number and operations content standard provided student work showing unusual solutions. One of these number and operation items centered on the fraction problem solved by Shanna, noted above, where the student used a mathematically valid approach, yet alternative to what the teachers might have been accustomed to seeing. For this item, seventeen out of twenty teachers did not accept this as a valid method, insisting that the student should have used the “invert and multiply” strategy. One other teacher responded that the student “got lucky.” The second number and operations item involved a three-digit subtraction problem for which the student, Mike, developed his own alternate algorithm. See Figure 3.

Students are asked to solve the word problem:
Candy has 105 jelly beans, she eats 18 of them,
and how many does she have left? (The teacher
walks around and sees a variety of answers,
including Mike’s.)

Mike’s work

$$\begin{array}{r}
 105 \\
 -18 \\
 \hline
 3 \\
 90 \\
 \hline
 87
 \end{array}$$

Figure 3. Mike’s Jelly Bean Problem Work

Thirteen of the twenty teachers refused to accept this student’s mathematically valid solution as a legitimate solution. In both of these number and operations items, the teachers who doubted the solutions may or may not have lacked mathematics content knowledge, but in any case were unwilling to accept the alternative strategies. This suggests a reluctance to value student thinking, which would hinder the use of a Learner-Responsive pedagogical approach.

Two items in the sample drawn on the data analysis and probability standard also revealed interesting findings. One item included student interpretation of a graph that had no labels or numbers. See Figure 4.

Students were asked to tell a story to go
with the graph below.
Maris’ story was about a sailboat speed in a
race.

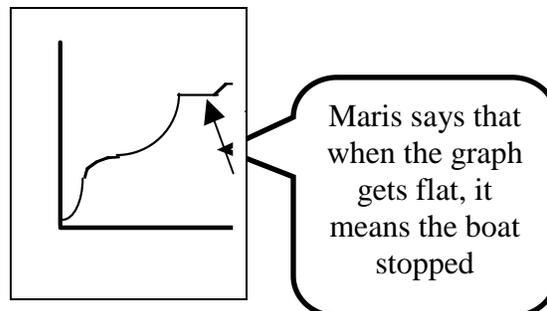


Figure 4. Maris’ Graphing Problem Solution

The student explanation described a representation of distance against time, but every teacher in the sample of 20 viewed the graph as representing only speed against time. Hence no teacher interpreted the student's explanation as correct.

A second problem drawn on the data analysis and probability standard, focusing on probability is inserted below:

Students in 2 fourth grade classrooms were examining bags of Skittles and looking at the distribution of colors. In one classroom the median number of green Skittles in the bags was 9. In the other classroom the median number of green Skittles in the bags was 7. Sabrina concluded that the classrooms did not use the same size bags of Skittles.

Twenty percent (20%) of the teachers participating in this research and professional development, responded with thorough explanations revealing an understanding of the mathematics; but most of the remaining teachers offered responses that were clearly incorrect or with what we might call "non-answers," circumventing the topic and suggesting little to no knowledge of the relevant content.

In both cases of teacher responses to the probability problem, data suggest that even those with overall Problem/Activity-Based or Student-Centered (PSC) or Learner-Responsive Pedagogy (LRP) pedagogical perspectives did not know this particular mathematics well enough to question students through explorations or help students come to a mathematically valid understanding.

5. Closing Discussion

As noted earlier, the professional development context for this research study has among its goals to support teacher movement in mathematics content perspectives and mathematics pedagogy perspectives. Although we found that the professional development experiences impacted teacher perspectives in some areas, but had mixed results overall. The more important finding in this work, however, is the utility of our coding analysis in documenting and describing teacher movement. That we can capture even small movement suggests a useful methodology in capturing the subtleties of incremental change. Additionally, as opposed to definitive, consistent, permanent positions, teachers are positioned in-between categories on our continua, tend toward a position, or contribute data that shows only slight movement toward a position. In the on-going work many of us do with teachers, being able to identify subtle changes and the nuances of individual teacher's perspectives is critical to our work. That a set of teachers may fill in many different positions on the continua does not suggest more codes are needed, but, rather, that the continua represent the realities of teacher growth. They are practical and useful tools for describing fluid movement, being flexible enough to capture the teachers' sometimes daily and often small, incremental changes in perspectives. Many teachers also are likely to be positioned differently for some content than for others, so connecting this work to teacher content knowledge can reveal additional directions for professional development.

Finally, this work has implications for equity pedagogy (Erchick, Dornoo, Joseph, and Brosnan 2010). A teacher's strict and superficial procedural perspective on mathematics limits students' opportunities for the rich mathematical learning of the integrated procedural/conceptual perspective; and examples of limitations of content knowledge that emerged in this work also hinder students' access to the mathematics. A teacher directed perspective as we define it in this study is akin to Friere's "banking" concept of education" (1973; 1989, p. 58), where the teacher transmits, deposits, and "the scope of action allowed to the students extends only as far as receiving, filling, and storing the deposits" (p. 58) and does not allow for the shared authority, and the accompanying learning, that is necessary for Learner-Responsive Pedagogy.

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