

## On Modeling and Estimation of Travel Behavior Utilizing Varying Contextual Information

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### Abstract

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*This paper attempts to outline the essence of a methodological approach which allows the researcher to utilize relevant contextual information in modeling and estimation of travel behavior in a manner that takes account of the varying context. For this purpose, we utilize a varying coefficients model to explain travel behavior, using cross-country data from time-use studies. This study goes beyond previous studies in two directions. First, it examines travel behavior as occurring in a broader context by considering a number of contextual elements which take on meaning within the activity systems framework. Second, as far as our model and test procedure are concerned, our analysis adopts a model which can more correctly relate the response variable to its determinants and also permits the impact of those determinants to be country/group specific. Our empirical results appear to indicate that travel behavior is the product of a variety of forces captured in several contextual dimensions.*

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**Keywords:** Travel Behavior Analysis, Cross-Country Time-Use Data, Varying Coefficients Model

### Introduction

The study of activity systems strives to develop both analytical and empirical base for modeling daily behavior in general and travel behavior in particular. Typically in studying travel behavior the dimensions studied relate either to the traveler or to the environment in which the traveler acts. Pioneering work on the activity-based approach was conducted at the Transport Studies Unit at Oxford University, see Jones (1979). The Oxford group undertook a project of theory-building checked and bolstered by empirical analysis. The project in essence bridged the gap between the Haggard (1970) time space approach and the more familiar travel survey approach. According to Pas & Harvey (1997), the activity-based approach to travel demand analysis spans a variety of theoretical and methodological approaches. Themes recurring in this work include: (1) analysis of travel as a derived demand; (2) the scheduling of activities in time and space; (3) the spatial-temporal and interpersonal constraints on activity and travel choice; (4) the interactions between different persons and between activity and travel decisions over the course of a day or a longer time period; and (5) the structure of the household. The major difference between the activity-based approach and the trip-based approach is in the treatment of time. In a recent paper on emerging issues in travel behavior analysis, Pendyala and Bhat (2013) supported “... move towards an activity-based time use survey format incorporating questions about attitudes, perceptions, values, information acquisition and use, and decision making processes.” For more on travel behavior analysis, see for instance Cervero and Hansen (2002), Cervero, R. (2003), Clifton and Handy (2003), and Bhat, Guo, and Srinivasan, and A. Sivakumar (2004), Pendyala and Bhat (2008), Pinjari and Bhat (2011), and Pendyala et al (2012).

## Daily Activity & Travel Behavior

The foregoing suggests the elements of the framework within which daily activity and travel behavior must be analyzed. Additionally it is necessary to identify the elements of the activity system of major interest to modeling. These elements are actors, activities, time, space, and social contact.

Actors are simply individuals whose spatial-temporal activities are being considered. The significance of sex and employment status; and the presence of young children in determining the activity patterns of individuals pervade the time-use/travel literature. See, for instance, Clark and Harvey (1976), Robinson, Kitamura and Golob (1992), and Chapin (1974). Given the constraints imposed by an individual's place in the family structure (primarily sex and life cycle determined) and the society work structure, approximately three quarters of all activity episodes may be routine, see Cullen (1975). It is generally most fruitful to begin an analysis of behavior patterns by focusing on groups primarily in terms of life cycle. In the current analysis, life cycle is expressed in terms of variables for age and marital status.

Activity definition and identification are difficult in that activities are multidimensional. A particular problem is the changing nature of a given activity in the light of the context within which it is undertaken. For example shopping can easily be either a work or leisure activity. Serious activity analysis will continue to suffer until appropriate taxonomic methods or schemata are developed. To date the coding approach of the multinational time budget project has been the only one to provide any order, see Szalai (1972). That system, however, must be collapsed into a more workable structure for planning purposes. The analysis presented here focuses on a limited number of activities, trips, work and sleep.

Two primary dimensions of time, which are relevant to behavioral modeling, are incorporated into the analysis here, namely position and duration. Position represents the point at which actions occur (for example, weekday or weekend, morning or evening) represents position. The period during which actions occur-minutes are used as the duration metric in this report- and direction. The work of Pas (1988) has shown the importance of treating time at several levels. Working with week-long diaries he identified a two stage process where travel was determined first at the weekly level and second on a daily basis. With this, and other work, he has shown that multiple diaries for a respondent reduce the variance of measured travel behaviour. However, the analysis presented here deals only with data treated as single day diaries. Harvey (1978), studying the effect of a broad range of background variables on discretionary time observed that "only characteristics of the day [workday, Sunday, Holiday] as a group exceed the overall average. This suggests a strong structuring of daily time-use by forces which fall outside the personal or household characteristics of the respondent." This factor is represented here using the temporal position variable "type of day." Two duration variables are used in the analysis, the average daily duration of sleep and the duration of time spent travelling.

**Space** has several distinct aspects of relevance. The analysis here incorporates only adapted space in considering the location of paid work. Adapted space is concerned with locations which have, through design, development or mere habitual use, become the sites of continual, regular or recurring activities, i.e. offices, shopping centers, parks.

Behavioral modeling must account for behavior in terms of each of the elements discussed above. At one level, the interaction of all of these elements is manifest in individual behavior, while at another level it is manifest in the patterns of the whole. According to Buckley (1967), "the total act involves an 'impulse', or problem induced tension, or goal-in-view, as well as the person's selective perception and manipulation of the environment, such that each of the elements is defined or given meaning only in terms of the others." On the other hand, Kutter (1973) argued that "the spatial and temporal activity pattern is determined by the concurrence of individual activity patterns."

The task of the modeler is to account for behavior in a way that allows one to understand and utilize changes in it for policy guidance and evaluation. One recent study provides some insight into the approach typically taken to understand the forces acting on travel behavior. It explores the relationship between demographic and socioeconomic forces, time allocation and their implications for travel, as well as theory related to time money tradeoffs, see Levinson and Kumar (1995). It then undertakes a regression analysis to "quantify the factors affecting time allocation in 1988 for home, shopping and other activities" (Levinson and Kumar, p.459). For more about modeling travel behavior, see Al-Jammal and Parkany (2003).

The study here goes beyond such an approach in two directions. First, as discussed above, it examines travel behavior as occurring in a broader context than the demographic socio-economic context adopted by Levinson and Kumar (1995) by considering a number of contextual elements which take on meaning within the activity systems framework. Additionally, this paper illustrates the application of an important methodological approach to modeling and estimation - the varying or random coefficients model, in the context of the study of activity systems. More specifically it utilizes a varying coefficients approach to explain travel behavior as occurring in a context by considering a number of contextual elements using data from time-use studies collected in Austria, Canada, Norway, and Sweden.

**The Data, the Model, and the Test Procedure**

The data used in this study consists of random samples of sizes between 347-448 individuals selected from the 1990s time diary data sets collected in Austria, Canada, Norway, and Sweden. Table 1 presents some statistics about the number of trips, duration of travel, and duration of each trip in these four countries.

The data speak for themselves. So far as the averages of number of trips and duration of each trip are concerned, the reported F statistics, computed from the analysis of variance (ANOVA) procedure, show that real differences exist among the four countries. However, in terms of the average duration of travel the four countries did not differ significantly. The computed coefficient of variations also point out that the Austrian data show more variation so far as the duration of each trip is concerned but less variation than other countries in terms of number of trips. Indeed, Swedish data show less variation in the other two cases.

So far as our model and test procedure are concerned, our starting point is the following multiple regression model:

$$Y_{ij} = \alpha_1 + \alpha_2 X_{2ij} + \alpha_3 X_{3ij} + \alpha_4 X_{4ij} + \alpha_5 X_{5ij} + \alpha_6 X_{6ij} + \alpha_7 X_{7ij} + \alpha_8 X_{8ij} + \alpha_9 X_{9ij} + \alpha_{10} X_{10ij} + u_{ij} \quad (1)$$

where Y = number of trips,

- X<sub>2</sub> = duration of travel (in minutes),
- X<sub>3</sub> = duration of essential sleep (in minutes),
- X<sub>4</sub> = age (less than 14 (1), 15-24 (2), 25-44 (3), 45-64, 65 and over (4))
- X<sub>5</sub> = marital status(single (0), married (1)),
- X<sub>6</sub> = gender (male (0), female (1))
- X<sub>7</sub> = pet (have (1) or have not (0)),
- X<sub>8</sub> = number of shopping trips,
- X<sub>9</sub> = work location (do not work(1), home (2), work place (3), other (4))
- X<sub>10</sub> = type of day (weekday (0), weekend (1)).

The *u*'s are the random disturbances and the subscript *i* (i=1,2,3,...,1635) indexes the individuals and the subscript *j* (j=1,2,3,4) refers to the countries. The model uses activity times (travel duration, duration of essential sleep), demographic characteristics (age, marital status, gender), and context of activities (pet, number of shopping trips, work location, type of day) to explain the number of trips and travel behavior.

There are essentially two major difficulties associated with the above fixed coefficients model specification. First, the assumption that the parameters are the same across these four countries may be unrealistic given the cultural and social differences between these countries. Second, the regression model (1) would not represent the law relating the response variable Y to its determinants if the latter were correlated with the disturbance term (u). In particular, if the disturbance is viewed as a linear combination of excluded variables that along with the included variables is sufficient to determine Y, neither the slope coefficients nor the disturbance term in (1) are unique, and the former cannot be viewed as measuring the effects on Y. Furthermore, even an instrumental variables approach would not help in that if the included variables are not genetically independent of the disturbance term, these estimators would, in general, be inconsistent.

In light of the above problems, we approach the problems of specification and estimation in the following manner. Following AmirKhalkhali and Dar (1993), we assume that

$$Y_{ij} = \alpha_1 + \alpha_2 X_{2ij} + \alpha_3 X_{3ij} + \alpha_4 X_{4ij} + \alpha_5 X_{5ij} + \alpha_6 X_{6ij} + \alpha_7 X_{7ij} + \alpha_8 X_{8ij} + \alpha_9 X_{9ij} + \alpha_{10} X_{10ij} + W_{ij}'\delta \quad (2)$$

where W is the set of excluded variables that along with those that are included are sufficient to determine Y. However, in the linear, deterministic law stated by (2), neither the slope coefficients nor W are unique in that they are sensitive to the parameterization chosen. To ensure uniqueness, we postulate

$$W_{ij} = \gamma_{1j} + \gamma_{2j}X_{2ij} + \gamma_{3j}X_{3ij} + \gamma_{4j}X_{4ij} + \gamma_{5j}X_{5ij} + \gamma_{6j}X_{6ij} + \gamma_{7j}X_{7ij} + \gamma_{8j}X_{8ij} + \gamma_{9j}X_{9ij} + \gamma_{10j}X_{10ij} + v_{ij} \quad (3)$$

Substituting (3) into (2) yields

$$Y_{ij} = \beta_{1j} + \beta_{2j} X_{2ij} + \beta_{3j} X_{3ij} + \beta_{4j} X_{4ij} + \beta_{5j} X_{5ij} + \beta_{6j} X_{6ij} + \beta_{7j} X_{7ij} + \beta_{8j} X_{8ij} + \beta_{9j} X_{9ij} + \beta_{10j} X_{10ij} + \varepsilon_{ij} \quad (4)$$

with  $\beta_{1j} = \alpha_1 + \gamma_{1j}'\delta$ ,  $\beta_{2j} = \alpha_2 + \gamma_{2j}'\delta$ ,  $\beta_{3j} = \alpha_3 + \gamma_{3j}'\delta$ ,  $\beta_{4j} = \alpha_4 + \gamma_{4j}'\delta$ ,  $\beta_{5j} = \alpha_5 + \gamma_{5j}'\delta$ ,  $\beta_{6j} = \alpha_6 + \gamma_{6j}'\delta$ ,  $\beta_{7j} = \alpha_7 + \gamma_{7j}'\delta$ ,  $\beta_{8j} = \alpha_8 + \gamma_{8j}'\delta$ ,  $\beta_{9j} = \alpha_9 + \gamma_{9j}'\delta$ ,  $\beta_{10j} = \alpha_{10} + \gamma_{10j}'\delta$  and  $\varepsilon_{ij} = v_{ij}'\delta$ .

Note that (4) is a varying coefficients model, and that the disturbance is not the joint effect of excluded variables; instead, it is the joint effect of the remainder of the excluded variables after the effect of included variables has been factored out. Note also that whereas, the included variables cannot be uncorrelated with every variable that affects Y, they can be uncorrelated with the remainder of every such variable. This random coefficients Approach can be viewed as a refinement of laws as stated by Pratt and Schlaifer (1984, 1988). Accordingly, this model more correctly relates the response variable with respect to its determinants and also permits the impact of those determinants to be country-specific.

### The Empirical Results and Concluding Remarks

The above model is estimated by applying Swamy (1970), Swamy and Mehta (1975) and Swamy and Tavlak (2002) random coefficients generalized least squares (RGLS) estimator for the entire sample, as well as for each country. Note that, this method of estimation also address the important issue of heteroscedasticity that could be present in the country-specific cross-sectional data. A brief discussion of the details of the particular random coefficients technique employed in this study is contained in the Appendix. We also computed the Swamy's g-statistic (see the Appendix) to test for the validity of the random coefficients model. The results are presented in Table 2. For the pooled sample, the results indicate that shopping, working but not in a specific location, and pet care are the statistically significant contributors to the number of trips. In addition, those who spent more time on the roads, made more trips as well. However, aging and being married appeared as two constraints which are highly significant. The other two statistically significant constraints are the duration of essential sleeping and type of the day. The contribution of type of day to trip generation can be attributed to the location of work variable which essentially distinguishes weekdays from weekends.

So far as the country-specific results are concerned, in all four countries, shopping, pet times, and duration of travel continue to have a statistically significant impact on the number of trips. Essential sleep is again a significant constraint. In regard to other variables, there are differences among these countries. Aging is a significant constraint to trip generation in Canada, Norway, and Sweden but not in Austria. However, Austria is the only country where marital status affects significantly the number of trips, in that married persons made less trips. Gender plays a significant role in trip generation in Austria and Norway, in that males made more trips. Type of day is not a statistically significant variable in the case of Sweden. However, in other countries it affects the number of trips, in that people made less trips in the weekends. The reported  $R^2$ , computed as the square of the correlation between the actual and fitted values, show a reasonably good explanatory power of the estimated models in general, and in the cases of Sweden and Canada in particular. The computed value of g-statistic (GSTAT) is highly significant at the 5% level and clearly supports the random coefficients approach to differentiate the impacts of activity times, demographic characteristics, and context of activities on the number of trips and travel behavior in the four countries.

In conclusion, our analysis goes beyond previous studies by adopting the random coefficients model that can more correctly relates the number of trips and travel behavior with respect to its determinants and also permits the impact of those determinants to be country-specific. So far as our empirical results are concerned, as hypothesized, trip generation is the product of a variety of forces which can be captured in several contextual dimensions.

**Table1: Number of Trips, Duration of Travel, and Each Trip**

Country	Austria	Canada	Norway	Sweden	F statistic
<b>Number of Trips</b>					
Median	4.00	5.00	5.00	5.00	12.09*
Average	4.57	5.22	5.11	5.76	
Coefficient of Variation	0.45	0.59	0.51	0.49	
<b>Duration of Travel</b>					
Median	60.00	60.00	60.00	70.00	0.21
Average	76.38	76.99	75.71	80.13	
Coefficient of Variation	0.86	0.83	0.92	0.69	
<b>Duration of each Trip</b>					
Median	15.00	12.50	15.00	12.00	3.73*
Average	18.88	16.04	17.02	15.18	
Coefficient of Variation	1.05	0.78	0.96	0.76	
<b>Number of Observations</b>	348	434	404	449	

\*:Significant at the 5% level.

**Table 2: Number of Trips and Travel Behavior: GLS Results**

**The Model:**  $Y_{ij} = \beta_{1j} + \beta_{2j} X_{2ij} + \beta_{3j} X_{3ij} + \beta_{4j} X_{4ij} + \beta_{5j} X_{5ij} + \beta_{6j} X_{6ij} + \beta_{7j} X_{7ij} + \beta_{8j} X_{8ij} + \beta_{9j} X_{9ij} + \beta_{10j} X_{10ij} + u_{ij}$

Countries	Constant	Activity Time		Demographic Characteristics			Context of Activities				R <sup>2</sup>
	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	$\beta_6$	$\beta_7$	$\beta_8$	$\beta_9$	$\beta_{10}$	
<b>Austria</b>	6.40*	.007*	-.005*	-.041	-.722*	-.487*	.471*	.698*	.120	-.307*	<b>.26</b>
<b>Canada</b>	5.11*	.017*	-.002*	-.565*	-.142	-.031	.973*	.639*	.510*	-.659*	<b>.43</b>
<b>Norway</b>	5.51*	.011*	-.002*	-.193*	-.033	-.352*	.505*	.872*	.526*	-.704*	<b>.23</b>
<b>Sweden</b>	4.46*	.022*	-.002*	-.322*	-.072	.001	.649*	.568*	.319*	-.024	<b>.47</b>
<b>All</b>	5.37*	.014*	-.003*	-.280*	-.242	-.217	.649*	.694*	.369*	-.424*	<b>.34</b>
<b>GSTAT = 127.14*</b>											

where Y = number of trips, X<sub>2</sub> = duration of travel (in minutes), X<sub>3</sub> = duration of essential sleep (in minutes), X<sub>4</sub> = age, X<sub>5</sub> = marital status, X<sub>6</sub> = gender, X<sub>7</sub> = pet, X<sub>8</sub> = number of shopping trips, X<sub>9</sub> = work location, X<sub>10</sub> = type of day.

## References

- Al-Jammal, R. and E. Parkany (2003). Integration of Qualitative and Quantitative Methodologies: Framework and Quick Examples. *Transportation Research Record* 1854, *Journal of the Transportation Research Board*, National Research Council, Washington, D.C., pp. 171-179.
- AmirKhalkhali, S. and A. Dar (1993). Testing for Capital Mobility: A Random Coefficients Approach, *Empirical Economics*, 18, 523-541.
- Bhat, C. R., J. Y. Guo, S. Srinivasan, and A. Sivakumar (2004). A Comprehensive Econometric Microsimulator for Daily Activity-Travel Patterns, *Transportation Research Record*, Transportation Research Board, National Research Council, Washington, D.C.
- Cervero, R. (2003). Road Expansion, Urban Growth, and Induced Travel: A Path Analysis. *Journal of the American Planning Association* 69(2), pp. 145-163.
- Buckley, W. (1967). *Sociology and Modern Systems Theory*. Englewood Cliffs, N.J., Prentice Hall.
- Cervero, R. and M. Hansen (2002). Induced Travel Demand and Induced Road Investment: A Simultaneous Equation Analysis. *Journal of Transport Economics and Policy* 36(3), pp. 469-490.
- Cervero, R. (2003). Road Expansion, Urban Growth, and Induced Travel: A Path Analysis. *Journal of the American Planning Association* 69(2), pp. 145-163.
- Clifton, K. J. and S. L. Handy (2003). Qualitative Methods in Travel Behavior Research. In P.R. Stopher and P. Jones (eds.) *Transport Survey Quality and Innovation*. Elsevier Science Ltd, Oxford, UK, pp. 283-302.
- Chapin F. S., Jr. (1974). *Human activity patterns in the city: What do people do in time and space*. Toronto, John Wiley.
- Clark, S. and A.S. Harvey (1976). The sexual division of labour: The use of time. *Atlantis* 2(1):44-66.
- Cullen I. And V. Godson (1972). Networks of urban activities: The structure of activity patterns. Research paper No.1, University College, London School of Economics.
- Hagerstrand, T. (1970). What about people in regional science? *Papers and Proceedings of the Regional Science Association*, 24, 16-45.
- Harvey, A.S. (1978). Discretionary time activities in context. Occasional paper No. 3. Halifax, Canada: Institute of Public Affairs, Dalhousie University.
- Jones, P. M. (1983). *Understanding travel behaviour*. Great Britain, Antony Rowe Limited.
- Jones, P. M. (1979). New approaches to understanding travel behavior: The human activity approach. In Hensher, D. & Stopher, P. (Eds.), *Behavioral travel modelling*. London, Croom-Helm, 55-80.
- Kutter, E. (1973). A model for individual travel behaviour. *Urban Studies*, 10 (2), pp. 235-258.
- Levinson, D. and A. Kumar (1995). Activity, Travel, and the Allocation of Time. *Journal of the American Planning Association*. 61(4):458-470.
- Pas, E. & A.S. Harvey (1997). Time-use research and travel demand analysis and Modelling. In Ryuichi Kitamura and Martin Lee-Gosselin. *Understanding travel behavior in an era of change*. (Eds.). New York: Pergamon Press.
- Pendyala, R.M., C.R. Bhat, K.G. Goulias, R. Paleti, K.C. Konduri, R. Sidharthan, H.-H. Hu, G. Huang, and K.P. Christian (2012). Application of Socioeconomic Model System for Activity-Based Modeling, *Transportation Research Record*, Vol. 2303, 71-80.
- Pendyala, R.M., and C.R. Bhat (2008). Validation and Assessment of Activity-Based Travel Demand Modeling Systems, *Conference Proceedings 42, Innovations in Travel Demand Modeling: Summary of a Conference*, Transportation Research Board, Volume 2, 157-160.
- Pendyala, R.M. and C.R. Bhat (2012). Meeting the Travel Behaviour Research Needs of an Evolving World," *Travel Behaviour Research in an Evolving World: Selected Papers from the 12th International Conference on Travel Behavior Research*, Chapter 1, 3-12, edited by R.M. Pendyala and C.R. Bhat, Lulu.com Publishers, USA.
- Pinjari, A.R., and C.R. Bhat (2011). Activity Based Travel Demand Analysis," *A Handbook of Transport Economics*, Chapter 10, pp. 213-248, edited by A. de Palma, R. Lindsey, E. Quinet, and R. Vickerman, Edward Elgar Publishing Ltd.
- Pratt J. W. and R. Schlaifer (1984). On the Nature and Discovery of Structure, *Journal of the American Statistical Association*, 79, 9-33.
- Pratt J. W. and R. Schlaifer (1988). On the Interpretation and Observation of Laws, *Journal of Econometrics*, 39, 23-52.

Robinson, J. P., R. Kitamura & T.F. Golob (1993). Daily travel in The Netherlands and California: a time diary perspective. Hauge Consulting Group.  
 Szalai, A. Ed. (1972). The Use of Time. The Hague, Mouton.  
 Swamy, P.A.V.B. (1970). Efficient Inference in a Random Coefficients Regression Model, *Econometrica*, 38, 311-23.  
 Swamy, P.A.V.B. and J.S. Mehta (1975). Bayesian and Non-Bayesian Analysis of Switching Regressions and of Random Coefficient Regression Models, *Journal of the American Statistical Association*, 70, 593-602.  
 Swamy, P.A.V.B., and G.S. Tavlak (2002). Random Coefficient Models. In: Baltagi, B.H. (Ed.) *Companion to Theoretical Econometrics*, Basil Blackwell, 410-428.

**Appendix**

The varying or random coefficients model used in this study may be specified as

$$(1) \quad y_i = X_i B_i + u_i \quad (i=1,2,\dots,N)$$

$$(2) \quad B_i = B + e_i$$

for the i-th cross-sectional unit where  $y_i$  and  $X_i$  contain observations on the dependent variable and  $K$  explanatory variables over  $T$  periods of time, respectively. The disturbances  $u_i$  and  $e_i$  are assumed to obey the following  $E(u_i)=0$ ,  $E(u_i u_i') = \sigma_{ii} I$ ,  $E(u_i u_j') = 0$  and  $E(e_i)=0$ ,  $E(e_i e_i') = \Delta$ , and  $E(e_i e_j') = 0$ . In other words,  $B_i$  can be regarded as a random response vector with mean  $B$  and covariance matrix  $\Delta$ . Equations (1) and (2) can be expressed together as

$$(3) \quad y_i = X_i(B + e_i) + u_i$$

Including all  $NT$  observations, we can rewrite (3) as

$$(4) \quad Y = XB + ZE + U$$

where  $Y$  is an  $NT \times 1$  vector of observations on the dependent variable,  $X$  is an  $NT \times K$  matrix of observations on the explanatory variables,  $Z$  is an  $NT \times NK$  block diagonal matrix of observations on the explanatory variables,  $E$  is an  $NK \times 1$  vector of random elements, and  $B$  is the  $K \times 1$  mean vector coefficient. The composite disturbance  $(ZE + U)$  has a block diagonal covariance matrix  $\Phi$  with  $i$ -th diagonal block given by

$$(5) \quad \phi_{ii} = X_i \Delta X_i' + \sigma_{ii} I$$

The generalized least squares (GLS) estimator of  $B$  is given by

$$(6) \quad B = (X' \Phi^{-1} X)^{-1} X' \Phi^{-1} Y$$

and the variance-covariance matrix of  $B$  is  $(X' \Phi^{-1} X)^{-1}$ .

Swamy (1970) shows that unbiased estimators for unknown variances  $\sigma_{ii}$  and  $\Delta$  are

$$(7) \quad \sigma_{ii} = u_i' u_i / (T - K), \quad \text{where } u_i = y_i - X_i b_i \text{ and } b_i = (X_i' X_i)^{-1} X_i' y_i.$$

$$(8) \quad \Delta = [(\sum b_i b_i' - \sum b_i \sum b_i' / N) / (N - 1)] - [\sum \sigma_{ii} (X_i' X_i)^{-1} / N]$$

Swamy and Mehta (1975) have developed the best linear unbiased predictor (BLUP) for  $B_i$  which can be obtained as an estimate of the mean coefficient  $B$ , plus a predictor for  $e_i$  as follows

$$(9) \quad B_i = B + \Delta X_i' (X_i \Delta X_i' + \sigma_{ii} I)^{-1} (y_i - X_i B).$$

The null hypothesis  $H_0: B_1 = B_2 = \dots = B_N$  can be tested by the Swamy's suggested  $g$ -statistic (GSTAT)

$$(10) \quad GSTAT = \sum [(b_i - D)' X_i' X_i (b_i - D) / \sigma_{ii}] \chi^2_{K(N-1)} \quad \text{where } D = (\sum \sigma_{ii}^{-1} X_i' X_i)^{-1} \sum \sigma_{ii}^{-1} X_i' X_i b_i.$$