

## Exploring Preservice Teachers' Conceptual Understanding of Algebraic Ideas: Linear Function and Slope

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### Abstract

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*This study investigated preservice teachers' conceptual understanding of algebraic ideas focused on two specific mathematical concepts: slope and linear function. A Diagnostic Content Knowledge Assessment (DCKA) instrument was purposefully designed to assess participating preservice teachers' relevant knowledge levels through both quantitative and qualitative data collection and analyses. It was found that participating preservice teachers possessed poor understanding on these two concepts: only 26% percent of participants were able to reach a benchmark of 80% correctness, paralleling the standard of the state's content knowledge requirement for teacher certification. Special attention was given to the qualitative results of follow-up interviews with eight low-performing preservice teachers. Their responses supported the quantitative finding and also shed light on many common misconceptions in linear function and slope: lack of solid understanding of basic mathematical concepts and failure in using varied approaches. In addition, directions for further research are discussed.*

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**Keywords:** teacher education, middle school level, algebraic ideas, conceptual understanding, subject matter knowledge

### Introduction

Large-scale studies show that American students have made significant progress in the international mathematics tests over the last decade, such as the Trends in International Mathematics and Science Study (National Center for Education Statistics, 2003, 2007, and 2011). However, American students are still not ranked among the top performance group in the world. In addition, the teaching strategies and knowledge structure of American mathematics teachers are fundamentally different from those in higher achieving countries (Hill, Rowan, & Ball, 2005; Ma, 1999; Stevenson & Stigler, 1992). The National Council of Teachers of Mathematics (NCTM, 2000) suggests that the key to the improvement of students' performance in a subject-matter domain, such as algebra, is not the creation of an ever more elaborate and fine-tuned set of procedures, but rather by changing the nature of instruction. In essence, the change of students' performance is grounded on the teacher's competence in relation to content knowledge and pedagogical content knowledge.

Studies have demonstrated that many preservice and in-service teachers have poor understanding of many mathematics concepts and processes they need to teach (Ball, 1988; Ball, Thames, & Phelps, 2008; Ma, 1999; Masingila, Olanoff, & Kwaka, 2012; Zaslavsky, Sela, & Leron, 2002). A number of studies have examined preservice and in-service teachers' cognition level in terms of basic concepts, such as decimals, fractions, and the relationship between perimeter and area. Only a few studies have explored preservice teachers' understandings in relation to linear function and slope. Ball (1988) and Zaslavsky et al. (2002) identified that preservice teachers possessed poor background with these concepts, but failed to provide a comprehensive image of teachers' understanding in terms of attributes of linear functions. In this study, the researchers not only investigated participating preservice teachers' general cognitive level on these two concepts, but also explored specific misconceptions and potential factors contributing to their weak performance in these two concepts through answering the research question: *What is the nature of preservice teachers' knowledge and understanding of slope and linear function?*

## Literature Review

### The Role of Teachers' Knowledge

Few people disagree that teachers make a direct influence on students' learning and achievement (NCTM, 2000). However, how the influence occurs and what kind of knowledge structure ensures quality teaching has been still under heated debate. By examining a number of previous studies (e.g., Eisenhart & Borko, 1993; Putnam, Lampert, & Peterson, 1990; Resnick, 1989), Borko and Putnam (1996) uncovered several shared themes in learning to teach: (a) the central role of knowledge in thinking, acting, and learning, (b) learning as an active, constructive process, and (c) knowledge and learning as situated in contexts and cultures (p. 673). Therefore, knowledge has been deemed as pivotal in teachers' professional development.

The crucial role of teachers' knowledge has been recognized and respected for a long time. Elbaz (1983) asserted, "The single factor which seems to have the greatest power to carry forward our understanding of the teacher's role is the phenomenon of teacher knowledge" (p. 45). Shulman (1985) proposed the same view, "to be a teacher requires extensive and highly organized bodies of knowledge" (p. 47). The National Council of Teachers of Mathematics (NCTM) also highlighted the significance of teacher knowledge in the *Principles and Standards for School Mathematics* (NCTM, 2000) when stating, "Effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies" (p. 17).

### Research Literature on teachers' Subject Content Knowledge

Effective teachers must know and deeply understand the mathematics they teach and should be committed to their students as learners of mathematics (NCTM, 2000). Researchers have asserted that the teacher's subject content knowledge is especially important when student's conceptual understanding is a central goal (Ball et al., 2008; Borko & Putnam, 1996; Greenberg & Walsh, 2008). Besides concepts, facts, and procedures, teachers' subject content knowledge also involves knowledge of organizing ideas, connections among ideas, and ways of mathematical thinking and reasoning.

Considerable studies about preservice and in-service teachers' content knowledge have revealed that many teachers are ill prepared for the content they are supposed to teach (Ball, 1988, 1990a, 1990b; Hill, et al., 2005; Ma, 1999; Masingila, et al., 2012). For example, Ball (1990a, 1990b) pointed to the fact that preservice teachers had weak understandings of basic mathematical concepts such as place value, division, fraction, zero, and slope. Greenberg and Walsh (2008) resonated similar arguments.

Despite extensive studies concerning preservice teachers' mastery of basic algorithms, not much research has been done for teachers' knowledge of more advanced algebraic ideas, such as the concepts of linear function and slope.

In one study investigating the idea of slope (Zaslavsky et al., 2002), a diverse group of participants were recruited with various cognitive levels: eleventh-grade students, preservice secondary mathematics student teachers, in-service mathematics teachers, mathematics educators, and research mathematicians. Zaslavsky et al. (2002) focused on whether participants could identify the properties of the same linear function in homogeneous and non-homogeneous coordinate systems. The results of the study indicated that only 24% of the total participants and 32% of the preservice teachers could identify the correct slopes. This study proved that people in general lack a sufficient knowledge of slope, but it failed to depict a comprehensive image of how participants, especially those preservice teachers, would solve problems related to slope and linear function.

This present study scrutinized and probed middle level preservice teachers' knowledge in terms of linear function and slope comprehensively via a modified teacher' content knowledge diagnostic survey and follow-up interviews. The intent was to not only to explore preservice teachers' knowledge and understanding of algebraic concepts, but also to provide support and policy initiatives designed to improve teacher preparation program.

## Method

### Participants

This study used the strategy of convenience sampling to select participants from a large state university in the southwestern United States. Forty-three undergraduate students enrolled in a middle school mathematics methods course participated in this study. All these preservice teachers had been accepted into a teacher certification program and would potentially work as a mathematics teacher in grades 4-8. All participants were required to complete 24 credit hours of mathematics courses prior to graduation in order to be qualified to teach mathematics at the middle school level. In this study, all these participants were naturally classified into two groups in accordance with their class schedules: Group I (Monday class) and Group II (Wednesday class). Each specific participant was designated by a particular numeric number within her/his group, for instance, S13(W) represents student 13 from the Wednesday group.

### The Diagnostic Content Knowledge Assessment (DCKA) Instrument

The DCKA instrument used in this study consisted of fourteen multiple-choices questions to test different domains of concepts of linear function and slope. The instrument was developed by adapting questions from a number of authoritative resources, such as The Praxis Series™ assessments, the Texas Examinations of Educator Standards (TEExES), and Diagnostic Teacher Assessments in Mathematics and Science (DTAMS) issued by the research center of the University of Louisville. All the items were carefully chosen to match the testing scope of this study and two expert professors in the field of mathematics education were used to assess the content validity of the instrument. A coefficient alpha = .75 indicates satisfactory reliability of the instrument.

### Data Collection and Analysis

In the present study, numeric data were collected through the administration of the Diagnostic Content Knowledge Assessment (DCKA) instrument to all participants. The 14 multiple-choice questions on the DCKA assessed the preservice teachers' mastery of content knowledge and were evaluated quantitatively. Each correct answer was scored as '1' point respectively; in contrast, an incorrect answer was counted as '0' point. The percentage of the individual's total score against the overall 14 points reflected the preservice teacher' subject matter knowledge level.

The holistic view of participants' knowledge of linear function and slope was further examined by sorting them into six specific domains testing varying attributes regarding these two concepts. Table 1 categorizes all questions into appropriate domains and provides insights in subsequent data analysis.

In order to explain and extend the quantitative findings, narrative data from the follow-up interviews with eight participants were collected after the analysis of test scores. Semi-structured interviews were used to unpack in-depth information related to the test results that were deemed impossible to be answered exclusively by quantitative data per se. Eight low-performing participants were asked to redo the challenging questions, while speaking out loud their thinking processes as much as possible. Specifically in this study, two typical questions were analyzed with regards to *Identify linear function* (domain 5).

## Results

### Quantitative Data

#### 1) Passing Percentages of Diagnostic Content Test

According to the teacher certification policy in the state, which participants were certifying, teacher candidates are required to accurately answer 80% of the questions on their content knowledge exam in order to be qualified as middle level mathematics teachers. Since the question items in this study were selected and modified from a number of authoritative resources comparable to the traditional teacher certification exam level (e.g., The Praxis Series™ assessments), all participants' results were also evaluated at an 80% (11 items) level of correctness to see whether the preservice teachers had sufficient content knowledge to be potentially certified (see Table 2). Only 26% percent of participating preservice teachers were able to reach 80% correctness of the test.

The result shown in Table 2 suggests that participating preservice teachers possessed a poor understanding of content knowledge in this particular content area, which may point to unsuccessful performance on the upcoming certification test and in subsequent teaching practice.

## 2) Individual Item Analysis

As aforementioned, 14 items were compiled into six domains with distinctive attributes in relation to linear function and slope. In order to pinpoint participants' deficiencies and misconceptions correspondingly, both individual items and clustered domain descriptive data reports were examined. Participants largely experienced problems within four domains (2, 4, 5 & 6): *Properties of linear function and slope*, *Identify linear function (perpendicular)*, and *Identify linear function (parallel)*, and *Identify points on the linear function*. In this study, two representative items (items 4 and 7) for domains 5 were extracted for further qualitative investigation based on two criteria: (1) these items were performed relatively poorly by many of the participants; and (2) the question could elicit understanding or misconceptions with basics.

### Qualitative Data

Simply speaking, two lines in a plane that do not intersect or meet are called parallel lines. In a Cartesian coordinate system, two parallel lines have the same slope. Geometrically, parallel lines are conceived as two lines sharing the same angle from the horizontal with different y intercepts.

To understand the participating preservice teachers' cognitive understanding in relation to parallel lines and corresponding properties, items 4 and 7 were selected for further investigation. Based on information in Table 3, participants performed better on item 4 than on item 7. Consequently, analyzing the nature of the questions was deemed important before the examination of the participants' understanding of the concept.

Through dissecting the participants' written work and interviews, items 4 and 7 were found to have commonalities in certain attributes and differences in others (see Figures 1 & 2). Commonalities for both questions consisted of: (1) assessing the participants' knowledge regarding slope of parallel lines and their capability to associate the linear function formula with particular points; (2) not providing the algebraic equations for the original lines, which demanded the determination of the equation; and (3) a similar level of difficulty since item 4 gives out explicit points on the parallel line whereas item 7 provides explicit points on the original line.

Differences between the two items were also manifested, including: (1) difference in direction: one negative versus one positive; (2) item 4 asked for a parallel line while item 7 translated the same line; (3) item 4 gave an exact point (6, 12) on the parallel line while item 7 provided implicit information for points on the new line; and (4) item 7 giving explicit points (-2, 0) and (0, 6) for the original line, but no such information on the graph of item 4. In conclusion, these two questions were similar in the difficulty level but distinctive in delineation. In subsequent paragraphs, participants' responses to each question were analyzed respectively.

### 1) Responses on Item 4

Being able to determine the slope of the line was the premise in answering this question. To solve this problem, preservice teachers could determine  $b$  by applying the formula  $(y_2 - y_1)/(x_2 - x_1)$  to two points identified from the graph, e.g. (-1, 1) and (0, -2); or they could eliminate choices A and C due to the negative slope direction shown on the graph and end up with a slope of -3, which was common to both answers B and D.

After identifying  $b = -3$ , students could either plug in 6 or 12 to ascertain the final equation. Alternatively, they could apply (6, 12) into the point-slope formula  $(y - y_1) = 3(x - x_1)$  to calculate the unknown equation directly. However, this alternative method was adopted by only one preservice teacher in this study despite its nature as the most explicit expression.

The results revealed that nearly half of the participants responded incorrectly to this question. Varied reasons are reported in subsequent paragraphs to address participants' relevant uncertainties and misconceptions.

#### (1) Unable to identify the slope of the line

Many of the preservice teachers were unable to pinpoint two accurate points on the line in order to identify the slope of the line. For instance, S3(W) wrote “-2/1 = slope” while the point (1, -2) was obviously not on the line. Two other students calculated that the slope equaled -1 only because the line crossed the point at (-1, 1).

At least two points are used to determine the slope of a line, but the participants appeared to be unfamiliar with the formula of using two points to define the slope,  $m = (y_2 - y_1)/(x_2 - x_1)$ , since they determined a line by only one point (-1, 1). This formula appears frequently in 8<sup>th</sup> grade mathematics textbooks.

### (2) Unable to identify the value of b by given point

Secondly, unlike those participants who had problem identifying the slope at the onset, some individuals had clear and accurate thoughts concerning the value of  $b$ . They may, however, have been unsure how to utilize the given point of (6, 12). The simple application of one point in a linear function with known slope became a striking puzzle for many of the teacher candidates. For instance, S4(M) and S6(M) circled both choices  $B$  and  $D$  as the answer since they were unsure which one was correct. S6(M) expressed, “I know that I should be using the slope intercept form, but I don’t remember [it].”

### (3) Lack of calculation proficiency

In addition, lack of calculation proficiency was uncovered as another prominent factor resulting in mistakes. For instance, both S18(W) and S10(W) committed some algorithmic errors (see Figure 3 and 4). For instance, S18(W) applied a particular point (6, 12) into Choices B and D to generate equations,  $12 = -3(6) + 30$  and  $12 = -3(6) + 42$ , with the purpose of verifying which equation was suitable. To remove the parenthesis,  $-3(6)$  became  $-36$  in her solution, a typical miscalculation occur among students at early phase of algebraic learning (Matteson, 2010). This particular preservice teacher seemed to misunderstand the meaning of the parentheses here as an operational sign indicative of multiplication and exhibited an insufficient understanding of algorithm and properties of operations on numbers.

While S10(W) tried to isolate the  $y$  variable on one side of the equal sign, she had difficulty dealing with signs of other items, a typical problem frequently occurring among middle school students (Knuth, Stephens, McNeil, & Alabali, 2006). The participants’ written work manifested the fact that quite a few preservice teachers lacked a well-developed sense of numbers and the properties of number operations.

### (4) Unfamiliarity with using appropriate formulas

Similar to what happened in item 8, a correct answer to item 4 did not necessarily guarantee an unquestionable knowledge base. Although some participating preservice teachers chose the correct answer for this question, their problem solving processes were fraught with mistakes or meaningless nonsense, such as unsuccessful trial and error strategies. In actuality, 4 out of 8 interviewees provided problematic solutions regardless of their correct final answers. These solutions are elaborated upon in the subsequent paragraphs.

S2(M) started with a calculation of  $m$ . She easily figured out  $b = -3$  by using ‘rise over run’ strategy. Next she tried to plug the point (-2, 0) into answer choice B,  $y + 3x = 30$ . She wrote down  $-2 + 3(0) = 30$  and recognized this equation as useless. She did not associate (-2, 0) as a point from the original line and having nothing to do with finding the new line. She then attempted to generate a relationship between  $\Delta y$  and  $\Delta x$  by using the slope formula. Since she had already determined  $m$  in her first solution attempt, she could have worked backwards to determine the solution. She wrote down,

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{-2 - y}{0 - x} = \dots$$

After pausing for a while, S2(M) declared, “Ok, I am lost.” When she could not come up with a solution during the interview, she was prompted by showing her work on the pre-assessment. She looked at her answer sheet and pondered it for a while. Finally, she remarked that the reason she chose answer C was because she saw the same digit from both the question narrative and answer C [the given point (6, 12) in the question and the equation in choice C,  $y - 3x = -6$ , consist of common digit 6]. Her answer selection was based on pure intuition with no meaningful deduction. Despite recording the correct answer, S2(M) lacked the conceptual knowledge needed for this problem. Instead, she displayed the use of a trial and error strategy to test points and formulas.

### *2) Responses on Item 7*

Item 7 had a dramatically low correct response rate of 39.5% in the pretest irrespective of its essence as simply a problem of parallel lines. To appropriately answer this question, one needed to be familiar with the meaning of translation – moving every point on the line in a specified direction with a constant distance. Algebraically, a translated line that is parallel to the original line has an identical slope.

In this question, given the original  $x$  and  $y$  intercepts as  $(-2, 0)$  and  $(0, 6)$ , these two points can be identified as  $(7, 0)$  and  $(9, 6)$  after the translation of 9 units to the right. In addition, the new  $y$  intercept can also be discerned at  $(0, -21)$ . One could solve this problem by applying any discerned point  $(x_1, y_1)$  on the translated line by the point-slope formula of  $y - y_1 = m(x - x_1)$  to work out the new linear function or he/she could calculate the value of  $b$  in  $y = mx + b$  based on values of  $x_1$  and  $y_1$  and construct a slope-intercept equation.

Almost one third of the participants exhibited no understanding of the meaning of translation (see Table 4). Within those who failed to translate the line and to identify key points, only 7 participants figured out the correct points. In other words, 19 out of 26 (73.08%) of false responses indicated a possible cluelessness in terms of translation. The subsequent paragraphs examined various error types in detail.

#### (1) False answer with no work on the translated line

One third of these preservice teachers (32.6%) apparently did not know or could not recall the meaning of translation. Some representative remarks were found in their work sheets, such as “I forgot how to do this”, “I know I should be using the function  $f(x)$  but don’t remember now”, or “I am honestly guessing on this one.” In the follow-up interviews, several participants articulated their concern and confusion in this regard. For instance, S14(M) stated,

I am trying to drag pictures in my head again. I don’t pick anything ...and I know it’s your move this way, cause you move to the right. But, I don’t know if it’s, how you count them. It throws me off.

To clarify her “move to the right”, the interviewer asked, “do they move the...I mean changing the angle to translate or just get the parallel...[parallel line]?” S14(M) thought for a second and responded that the slope did not change the angle. However, she still could not envision how the points on the line moved to the right. She explained,

... I am trying to figure out is what you are adding to get nine units. Like, how do you add to from the negative two over, or do you just ...? You know that’s why I am thrown off by. You can mark these two points over here [she marked down intersections  $(-2, 0)$  and  $(0, 6)$  on  $x$  and  $y$  axes], but I don’t know if you add these you can have the rise over the run, but it doesn’t really give you a good number. I think this is one answer I don’t know how to do ...

Her remarks suggested that S14(M) had a rough image in her mind that the translated line should be parallel to the original line without angle change. Nevertheless, she seemed to have no idea how particular points performed the movement and where they end up.

Similarly, S14(W) asserted that “the  $x$  and  $y$  points will change one way. Umm, the  $y$  intercept ... translate nine units to the right...” Pondering for a while, she gave up finding more information. From her words, it is difficult to judge whether she meant the original slope should move from  $(0, 6)$  to  $(9, 6)$  horizontally or the new slope should be drawn by simply adding 9 to the value of the  $y$  intercept. The conversation suggested that most of these participants had some generic sense of translation, but were unable to nail down specific points after translation.

#### (2) False answer with inaccurate point(s) on the translated line

Unlike S14(W)’s ambiguity on the value of  $y$  intercept, S7(W) explicitly illustrated her understanding of the  $y$  intercept after translation as shown in Figure 5.

Her work conveyed that translation meant simply an addition, a context-free operation. Several possibly misconceptions were discerned: (1) The point she identified as the original  $y$  intercept  $(0, -2)$  was divergent from the authentic point  $(0, 6)$  shown on the graph. Her point was a reversal of the coordinates for the  $x$  intercept. Based on her response, she may have misinterpreted the  $y$  intercept as the opposite of  $x$  intercept. This assumption could be confirmed by her false response to item 8, where she wrote that “ $b = x$  intercept and is  $> 0$ .” In other words, she held a misconception that the  $y$  intercept was  $x$  intercept in essence. (2) She inappropriately represented all translations by just adding a certain number. The changes of  $x$  and  $y$  intercepts after the translation should consider both directions moved and the relation between  $x$  and  $y$  (see Table 5) as conditions. For this item, in order to identify the  $x$  value after horizontal translation, S7(W) only needed to add  $k$  units ( $k = 9$  in the question) to the  $x$  value as  $(-2 + 9, 0)$  to get  $(7, 0)$ . However, since her intention was to solve for the new  $y$  intercept, this was complicated by the need to consider the impact of the slope. Therefore, she needed to calculate the point as  $(0, 3 \times 9 - 6)$  to get  $(0, 21)$ , rather than simply adding 9 units to the original  $y$  intercept.

For this question, given the fact the coefficient of  $x$  equals 3 while  $x$  moves nine units to the right, the  $y$  should move  $9 \times 3$  to the negative  $y$  axis. Unfortunately, only 1 out the 43 participants exhibited a correct, clear-cut and comprehensive interpretation on this concept.

Another representative misinterpretation of translation was displayed by S9(M)'s work. Despite the point  $(-2, 0)$  originally intersecting the  $x$  axis, S9(M) highlighted  $(9, 0)$  as the result of translating 9 units to the right (see Figure 6).

Initially, S9(M) correctly calculated the slope by using two points  $(0, 6)$  and  $(-2, 0)$  and write down  $\frac{am}{bn} = \frac{-6}{-2} = 3$ . In taking a further step to figure out the unknown equation, she problematically applied the point-slope formula as  $(y + y_1) = m(x + x_1)$  as opposed to  $(y - y_1) = m(x - x_1)$ , which is shown in Figure 7. Like S5(W) and other participants, S9(M) used this flawed formula in some problem solving situations without thinking through possible errors. If she moved  $(x + 9)$  to the left side of the equal sign, she would be aware that the equation turned into  $\frac{(y + y_1)}{(x + x_1)} = m$ , which is different from the authentic slope formula as  $\frac{(y - y_1)}{(x - x_1)} = m$ . Nevertheless, no participant picked up on this misconception.

### (3) False answer with correct point(s) on the translated line

As mentioned earlier, only 7 out 26 participants with problematic responses could accurately locate correct points on the line after translating 9 units to the right. Points identified were either  $(7, 0)$  or  $(9, 6)$ . Two further steps would help reach the desired answer: (1) using either  $(7, 0)$  or  $(9, 6)$  to verify their answer choice; or (2) using either  $(7, 0)$  or  $(9, 6)$  to form the point-intercept equation as  $(y - y_1) = 3(x - x_1)$ . No matter which means was adopted, participants should have known the relationship between a given point and the linear function, in other words, how to make use of a point to figure out the unknown  $y$  intercept. For example, at the beginning, S10(W) easily pointed out  $(7, 0)$  and claimed the answer as D based on her instinct. She said,

I don't know. I put it in my head here. [she plugged  $(7, 0)$  into D and thereby getting  $3(7) - 2, 21 - 2 = 19$ ] I don't know. This almost seems logical. First, 27 and 21 [the constants in choices A and C], I don't think they all have large numbers. Anything I will go with B and D just because they are smaller numbers and we shift over but it's seven and zero. It didn't go out away here somewhere...

S10(W) had no persuasive argument for her answer choice. She stated, "... they are ... D is negative. Just because they are still positive, but they are like 21, it's way down here.... I don't know. I just know on calculator..." Her words seemed to signify that she could not find a more convincing reason other than her instinctive preference of smaller numbers and keeping the intercept positive. Mathematical decisions should be based on proof and mathematical reasoning (NCTM, 2000). Intuitions at times tell us the direction to explore, but not be relied upon to reach the final conclusion.

Some other participants also used their instincts in proving their solutions, with faulty results. For example, S13(W) alleged the reason she chose  $f(x) = 3x - 21$  was because the constant,  $-21$ , was divisible by 7 [the new  $x$  intercept  $(7, 0)$ ]. When asked why she chose a negative constant, she could not articulate her thoughts and responded, "I don't know. I honestly don't remember... because you can look at it again, it's negative. So it would be here." This participant seemed to have no clue with how to utilize the known point to construct a new linear function.

### **Summary**

Through in-depth analyses on students' responses to questions associated with parallel lines and the concept of translation, a variety of problems and misconceptions were highlighted. First of all, several preservice teachers exhibited problems in calculation proficiency by making mistakes at the algorithmic level. Preservice and inservice teachers' calculating proficiency within algebraic concepts must be better investigated. Consequently, the study sheds light on the critical fact that a few preservice teachers may have problems in number sense as well as properties of operations, which may cast negative influences on their teaching practices and their students' mathematical acquisitions.

Secondly, the data analysis of items 4 and 7 revealed a prevalent phenomenon that some participants tended to misrepresent or misinterpret formulas.

As referred to earlier, several erroneously illustrated the point-intercept formula as  $(y + y_1) = m(x + x_1)$  or  $(y - y_1) = m(x - x_1) + b$ , as opposed to the correct form:  $(y - y_1) = m(x - x_1)$ . Still others thought that  $b$  meant the  $x$  intercept. These misconceptions could possibly be passed on to students.

Thirdly and most importantly, this study had uncovered the fact that a significant number of participating preservice teachers were unable to apply formulas or given conditions flexibly into varied contexts due to their lack of profound content knowledge in the examined domain. This conclusion was drawn based on students' poor performance in both items 4 and 7. Given that both items asked for a flexible application of the point-slope formula, the data analysis revealed that most participants were unable to correctly address these questions. Considering the test difficulty level, this result was far less than satisfactory.

## Discussion

In this study, the researchers attempted to investigate the essence of preservice middle school mathematics teachers' content knowledge of two algebraic concepts – linear function and slope. The quantitative data analysis revealed that only 11 (26%) out of 43 preservice teachers were able to meet a passing standard of 80% correctness in regards to understanding of linear function and slope. In other words, more than two-thirds of the participants were potentially incapable to teach the related algebraic concepts according to the state teacher certification stands. This quantitative result suggests that the majority of participants possessed a poor understanding of the content. The notion of understanding in this study is consistent with the definition in *Understanding by Design* as “transferable, big ideas having enduring value beyond a specific topic” (Wiggins & McTighe, 2005, p. 128). Being able to flexibly transfer and apply abstract ideas to various situations is the essence of understanding, which was manifested little among participants in this study. The quantitative findings were supported by the following qualitative analysis. The participants' lack of deep understanding of linear function and slope partially explains their inability to transfer big mathematical ideas among various problem situations, as well as their failure to apply different concepts and procedures in solving a sophisticated problem.

## Limitations and Suggestions for Future Research

In order to better interpret and capitalize on the results of the present study, possible limitations should be highlighted and taken into consideration for future studies. First, the small number ( $N = 43$ ) of participants had potential to reduce the statistical power. Given the small number of participants, it was difficult to report the data as significant or not. For future studies, additional participants should be recruited to ensure more precise and persuasive results.

Second, the follow-up interviews only focused on eight lower-scoring participants based on the diagnostic test score. This selection strategy may not provide sufficient information to paint a holistic picture with respect to all preservice teachers' content knowledge mastery. In future studies, participants performing at low, mediate, and high levels on the diagnostic test should be included in the interview process, in order to compare and contrast and to find out their commonalities and differences.

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## Appendix

**Table 1: Domains of Item Distribution**

Domain	Domain description	Test Item numbers
1	Identify the slope	10, 12
2	Properties of linear function and slope	5, 8, 13
3	Identify linear function by points	11, 14
4	Identify linear function (perpendicular)	1, 2
5	Identify linear function (parallel)	4, 6, 7
6	Identify points on the linear function	3, 9

**Table 2: Students' Passing Rate with 80% Correctness**

	Group I ( $n = 22$ )	Group II ( $n = 21$ )	Total ( $N = 43$ )
Students of 80% correctness	5	6	11
Passing rate (percentage)	23%	29%	26%

**Table 3: Item Analyses**

Correctness	Item 4	Item 7
Frequency ( $N = 43$ )	22	17
percentage	51.2%	39.5%

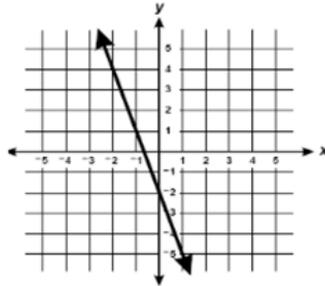
**Table 4: Student Performance Distribution on Item 7**

N = 43	Correct answer	False answer with no work on the translated line	False answer with false point(s) on the translated line	False answer with correct point(s) on the translated line
Frequency ( $N = 43$ )	17	14	5	7
percentage	39.5%	32.6%	11.6%	16.3%

**Table 5: Methods for Calculating X and Y Intercepts After Translation**

$y = mx + b$	Horizontally translating $k$ units	Vertically translating $k$ units
$x$ intercept	$(-b/m + k, 0)$	$(-(b+k)/m, 0)$
$y$ intercept	$(0, m \bullet k - b)$	$(0, b + k)$

Question 4: Use the graph below to answer the question that follows.



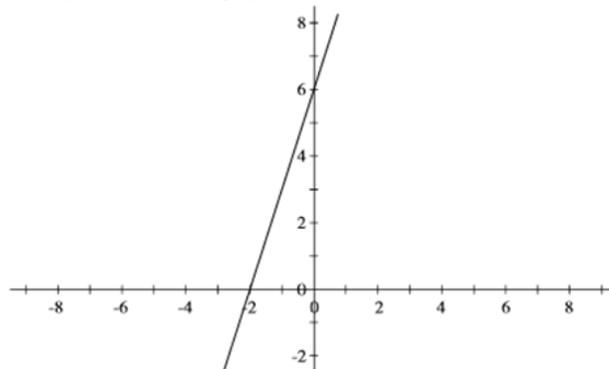
Which of the following lines has the same slope as the line above and passes through the point (6, 12)?

- A.  $y - 2x = 0$
- B.  $y + 3x = 30$
- C.  $y - 3x = -6$
- D.  $y + 3x = 42$

Correct answer: B

**Figure 1.** Test item 4 assessed participants' knowledge and application of attributes regarding parallel lines. Adapted from "TExES Preparation Manual - Mathematics 4-8." Copyright 2010 by Texas Education Agency.

Question 7: Find the function below which is the translation 9 units to the right of the function represented in the graph.



- a.  $f(x) = 3x - 21$
- b.  $f(x) = 3x + 14$
- c.  $f(x) = 3x + 27$
- d.  $f(x) = 3x - 2$

Correct answer: A

**Figure 2.** Test item 7 assessed participants' knowledge and application of attributes regarding parallel lines. Adapted from "Algebraic Ideas Assessment - Version 1." Copyright 2009 by University of Louisville Center for Research in Mathematics and Science Teacher Development.

4 Use the graph below to answer the question that follows.

Which of the following lines has the same slope as the line above and passes through the point (6, 12)?

~~A.  $y - 2x = 0$~~      ~~$y = 2x + 0$~~   
~~C.  $y - 3x = -6$~~      ~~$y = 3x - 6$~~

B.  $y + 3x = 30$      $y = -3x + 30$   
 D.  $y + 3x = 42$      $y = -3x + 42$

$12 = -3(6) + 30$   
 $12 = -36 + 30$   
 ~~$12 = -6$~~   
 $12 = -3(6) + 42$      $12 = -36 + 42$      $12 = 12$

**Figure 3.** S18(W)'s solution on item 4

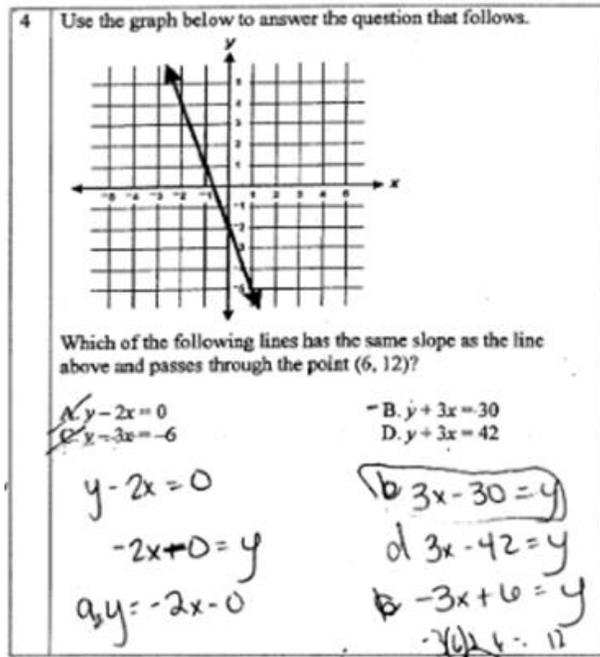


Figure 4. S10(W)'s solution on item 4

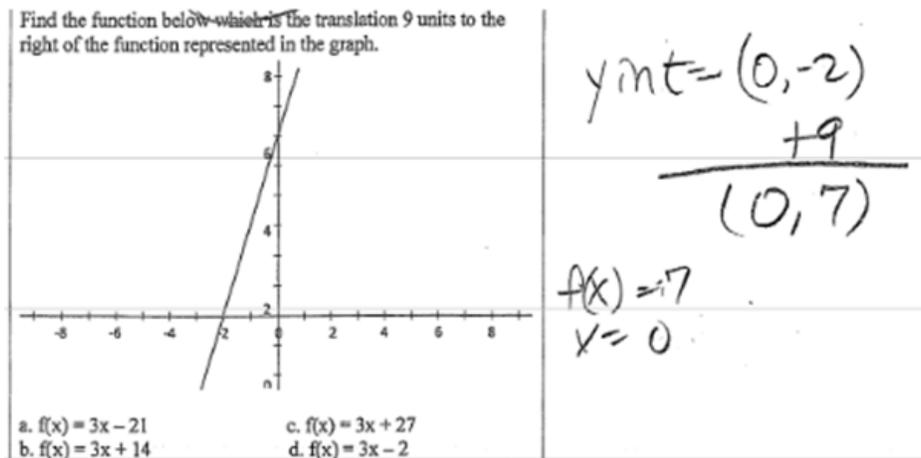


Figure 5. S7(W)'s response on item 7

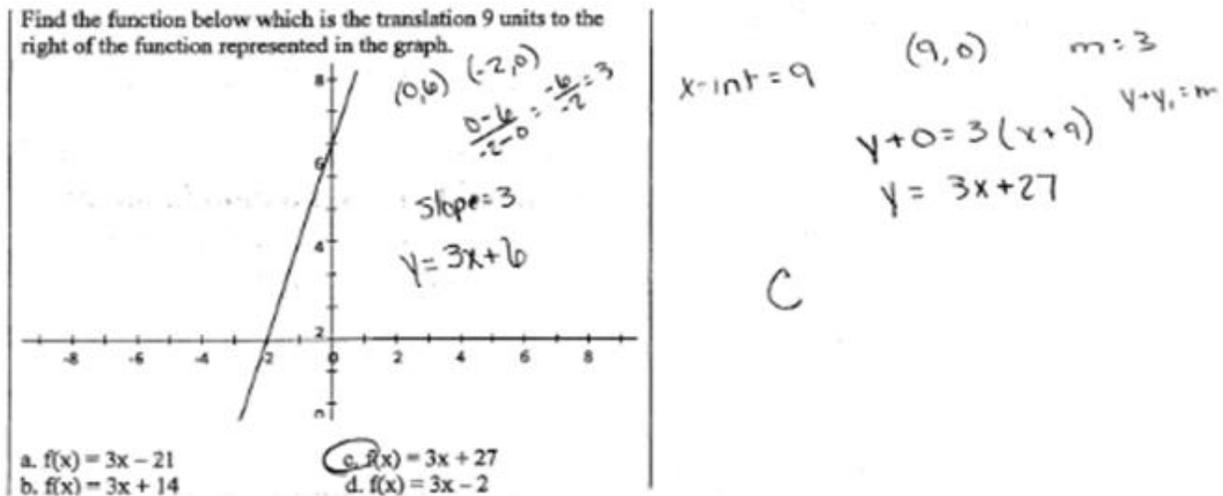


Figure 6. S9(M)'s response to item 7