Is the Diversification Benefit from Holding a Certain Number of Stocks a Reality or Myth?

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Abstract

Whether holding a certain number of stocks in a portfolio provides full diversification benefit in different market cycles is an important question for portfolio management. Portfolio managers are also interested to know whether any significant difference in the optimum number of securities for deriving full diversification benefit exists between naive portfolios and differently weighted portfolios. The study concluded that naive portfolios should hold between 9 to 15 stocks to get full diversification benefit. Surprisingly, the study did not find any definite number of stocks differently weighted portfolios should hold to get full diversification benefit.

Keywords: Naive Portfolio; differently weighted portfolio; risk diversification; diversified portfolio; and diversification ratio.

1. Introduction

Portfolio managers look for diversification benefit by distributing investments among different, less than perfectly correlated stocks, sectors or countries. A key question is how many stocks portfolios should hold to derive full diversification benefit. Vilija Alekneviciene (2012) showed the impact of portfolio size on portfolio diversification in Lithuanian Stock Exchange. Several other researches were also carried out in the field of portfolio risk diversification. Evans & Archer (1968); Statman (1987); Newbould & Poon (1993); Tang (2004); Solnik (1990) all studied the different possible ways of deriving diversification benefit. Evans and Archer (1968) showed that a diversified portfolio should hold a minimum of 10 securities. Statman (1987) argued that a diversified portfolio should include at least 30 stocks.

However, all of these researches did not show whether portfolios holding a certain number of stocks get full diversification benefit in different market cycles. The assumption that diversified portfolios hold a certain number of stocks will be plausible if it can be proved that the diversification benefits of fully diversified portfolios do not fluctuate significantly in different bull and bear market conditions. The study examined the variability in the optimum number of stocks of fully diversified portfolios in different bull and bears market conditions; and found out the optimum number of stocks portfolios should hold to get full diversification benefit in all market conditions. This study also compares the diversification benefits of naive portfolios and differently weighted portfolios in different market conditions.

2. Related Research

Several researches have been carried out on the diversification effect in portfolio management. While diversification is almost always beneficial in reducing portfolio risk, many people have argued over the degree of diversification benefit. Meir Statman (1987), in his study titled "How Many Stocks Make a Diversified Portfolio," argued that a diversified portfolio of randomly chosen stocks includes at least 30 stocks for a borrowing investor and 40 stocks for a lending investor. This argument is against the widely accepted concept that the benefits of diversification for stock portfolios are exhausted when the number of stocks reaches 10 or 15.

Woerheide and Persson (1993) in their "An Index of Portfolio Diversification" evaluated the ability of five different measures of diversification to provide meaningful information about the degree of diversification of an unevenly distributed stock portfolio. Being the best among these five measures, the complement of the Herfindahl index seemed to provide adequate explanatory power for general use. This study further shows that an index of portfolio diversification can be constructed and can be used by different parties to evaluate the degree of diversification of any stock portfolio.

On the other hand, Evans and Archer (1968) in their "Diversification and the Reduction of Dispersion: An Empirical Analysis" raised doubts concerning the economic justification of increasing portfolio sizes beyond 10 or more securities, and stressed the need for analysts as well as the private investors alike to include some form of marginal analysis in their portfolio selection models. They examined the rate at which the variation of returns for randomly selected portfolios is reduced as a function of the number of securities included in the portfolio.

More recently, Busarakamwong, Loong Ng and Stamenovic (2004) in their "A Framework for Assessing the Diversification Benefits of Additional Securities on Portfolio Risk" established a framework to assess the diversification benefits of adding more securities into a portfolio given different investor characteristics and assets involved. Their study proved that there is a strong positive relationship between targeted returns, risk appetite, base portfolio aggressiveness, and the benefit of adding securities to a portfolio.

In addition, Tang (2003), in his study "How Efficient is Naive Portfolio Diversification? An Educational Note," argued against the common knowledge that although portfolio diversification can help reduce investment risk without sacrificing the expected rate of return, the benefit of diversification is exhausted with a portfolio size of 10 to 15. He examined the issue on naïve (equal) weight diversification and analytically showed that for an infinite population of stocks, a portfolio size of 20 is required to eliminate 95% of the diversifiable risk, whereas such requirement for a portfolio size of 100 is 99%. In contrast, for a finite population of stocks, the corresponding portfolio size required is smaller, the smaller the population size.

Also Burnside (2004) in his "How Many Stocks Do You Need to be Diversified" had argued against the concept that assured diversification can be accomplished with a relatively small portfolio of common stocks and opined that in order to achieve the best diversification, stocks should be weighted by capitalization size.

Lee and Byrne (2004) "Risk Reduction and Real Estate Portfolio Size" studied the effect of portfolio size on risk reduction using actual property data over the period 1981 to 1996 and suggested the fund managers to be confident that their portfolio will have a risk level more like the average they need to hold portfolios of a considerably greater size than they might expect, or can sensibly hold. They concluded that size alone doesn't necessarily lead to a reduction in portfolio risk, while the other factors are given greater importance.

Besides, Solnik (1990) in his study "Swap Pricing and Default Risk: A Note" showed that the required markup to be charged over a specific amount for a default-free swap is a function of the subjective probability of default and of the value of default-free interest rate options. The study had further sorted out the relationship between the fair value of the swap markup and the yield spread on a straight loan to a similar risky client. The model he suggested only gives an upper bound on the above mentioned relationship.

Gabriel and Christof (2011) in their "On the Diversification of Portfolios of Risky Assets" introduced a measure of diversification for portfolios comprising the risky assets. They carried out their study on monthly return data for the S&P500 constituents, with a return history that spanned the last five decades. Their result concluded that a well-founded selection of assets would lead to a better portfolio diversification than that of a naïve allocation.

Most Recently, Alekneviciene (2012) "Portfolio Size and Diversification Effects in Lithuanian Stock Exchange" showed that naïve portfolios and differently weighted portfolios should have 24 and 27 stocks respectively to get full diversification benefit.

The study also showed strong relationship between investment concentration and portfolio risk for a differently weighted portfolio. It also found strong relationship between the number of stocks in a portfolio and portfolio risk for both naive and differently weighted portfolios.

Newbould and Poon (1993) surveyed a number of academic studies and US investment textbooks and find out the consensus view that a diversified portfolio should consist of more than 20 stocks.

Reilly and Brown (2010) mentioned in their book "Investment Analysis and Portfolio Management" that 90% of the diversification benefit can be achieved by holding 12 to 18 stocks in a portfolio.

Putting together the history of enormous debate regarding the adequate portfolio size and the apparent trend of stocks to contain increased diversifiable risk, it can clearly be concluded that an improved understanding of portfolio size and diversification is a need of the time.

3. Methodology

Thirty constituent companies of Dow Jones Industrial Average (DJIA) index as of March 2015 were used in this study. Monthly adjusted data from January 2001 to March 2015 were the basis of the study. One company, Visa (Sticker: V) of DJIA, was listed after January 2001. This company was excluded from the study to maintain the similarity in the time period of data of the companies included in the study. The variability in the optimum number of stocks of fully diversified portfolios will be assessed in four different market conditions:

- 1. Bear market 1: January 2001 to February 2003
- 2. Bull market 1: March 2003 to September 2007
- 3. Bear market 2: October 2007 to January 2009
- 4. Bull market 2: February 2009 to April 2015

Monthly returns of each individual stock were calculated.

Holding period return of individual stock,
$$r_{si} = \frac{P_1 - P_0}{P_0}$$
.....(i)

Where.

 P_1 = Stock price at the end of the period and

 P_0 =Stock price at the beginning of the period

Portfolio return is the weighted average return of the individual stocks in the portfolio. Here, the weight of the individual stocks corresponds to the portfolio exposure to the particular stock.

Portfolio return, $r_{pi} = \sum w_{si} r_{si}$(ii)

Where,

 w_{si} =Weight of the stock s in the portfolio and

 r_{si} = Holding period return of the individual stocks in the portfolio

Total risk (both systematic and unsystematic risk) was considered in this study. The study focused on the impact of the number of stocks included in a portfolio on the concentration of company specific risk (unsystematic risk) in that particular portfolio. Standard deviation of the stocks was used to measure the dispersion of returns around

their mean in a particular time period. Variance of stock return, $\sigma_s^2 = \sum_{i=1}^n \frac{(r_{is} - \bar{r})^2}{n-1}$(iii) Where,

 r_{is} = Return of stock;

 \bar{r} = Average return of stock and

n =Number of stocks

Standard deviation of stock return, $\delta_s = \sqrt{\sigma_s^2}$(iv)

Portfolio risk is a function of the variance of constituent stocks and covariance between each pairs of the stocks. The basic assumptions used in the portfolio risk calculation are that average return, standard deviation and covariance will remain unchanged and the distributions of monthly return data series are symmetric around the average return.

Standard deviation of the portfolio return:

$$\sigma_p = \sqrt{\sum_{i=1}^n \sigma_i^2 w_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_i \sigma_j Corr(r_i r_j)....(v)}$$

Where,

w= weight of stocks,

 σ = standard deviation of stocks and

 $Corr(r_i r_j) = Correlation between stock i and stock j.$

At this stage, two kinds of portfolio were formed in the analysis - naive portfolio and market capitalization weighted portfolio. Naive portfolios had the same weight for all the stocks included in the portfolio. In the market capitalization weighted portfolios, the weight of constituent companies was determined based on the market capitalization of each stocks included in the portfolio.

Weight of stock in the naive portfolio $w_i = \frac{1}{n}$(vi)

Weight of stock in the differently weighted portfoliow_i = $\frac{C_s}{C_n}$(vii)

Where,

 C_s =Market capitalization of each individual stock; and

 C_p = Total Market Value of the portfolio

Twenty-eight different portfolios were constructed from all twenty nine stocks. The underlying assumptions in the portfolio construction were as follows:

- 1. The first two-stock portfolio was formed based on lowest negative correlation
- 2. The subsequent portfolios were formed by taking the stock which has the lowest correlation with the existing portfolio.

At last, diversification effect of the increasing number of stocks in different market conditions was analyzed using diversification effect (DE) ratio. Vilija Alekneviciene (2012) in his "Portfolio Size and Diversification Effect in Lithuanian Stock Exchange Market" used DE ratio to analyze diversification benefit. DE ratio assumes value from 0 to 100. The higher the DE ratio of a portfolio, the more the diversification effect a portfolio has. Standard deviation of allocation only portfolios was required to calculate DE ratio. Allocation only portfolios assume zero correlation among the constituent securities.

DE ratio, $DE = 1 - \frac{\sigma_p}{\sigma_a}$(viii)

Where,

 σ_p = Standard deviation of the portfolio; and

 σ_a = Standard deviation of the allocation only portfolio

Standard deviation of the allocation only portfolio $\sigma_a = \sum w_i \sigma_i \dots (ix)$

Non-diversification risk was also taken out from the total portfolio risk to find out the real diversification effect. It was calculated as the lowest of all the standard deviations of the 28 portfolio combinations. DE ratio after excluding non-diversification risk from the total portfolio stands as

 $DE = 1 - \frac{\sigma_p - \min(\sigma_p)}{\sigma_a - \min(\sigma_p)}....(x)$

Where,

 $min(\sigma_p)$ = The lowest of all the standard deviations of the twenty eight portfolio combinations

4. Research Results and Interpretations

Table 1 illustrates diversification effect of the number of stocks included in both naive and differently weighted portfolios during the whole time period under study. The two-stock differently weighted portfolio had the higher diversification benefit than the two-stock naive portfolio (72.08 vs. 66.07). Around 72.08% of non-systematic risk went down in the two-stock differently weighted portfolio where only 66.07% of non-systematic risk eliminated in the two-stock naive portfolio.

For differently weighted portfolios, diversification benefit increased with the inclusion of more securities till portfolio 2. The portfolio having 3 stocks achieved full diversification benefit. Total non-systemic risk got eliminated at this level. Beyond this point, diversification benefit had always been below 100%. In naive portfolios, diversification benefit increased with the inclusion of more securities till portfolio 11. Naive portfolio consisting of 12 stocks provided full diversification benefit. Beyond this point, naive portfolios continued to lose diversification benefit with the inclusion of additional securities.

Table 2 illustrates that a definite conclusion cannot be made regarding the required number of stocks to be included in differently weighted portfolios to get full diversification benefit.

A portfolio consisting of 3 stocks got full diversification benefit during the period under study. However, a portfolio would lose diversification benefit significantly if it continued to hold 3 stocks in both bull and bear market condition. The authors also did not find any consistency in the number of stocks a portfolio should hold in either of the market conditions to get full diversification benefit. A fully diversified portfolio included 15 stocks in the first bear market condition and 10 stocks in the second bear market condition. On the other hand, a fully diversified portfolio included 5 stocks and 4 stocks in the first and second bull market condition respectively. Even diversification benefit of the same portfolio fluctuated considerably in two different bull market periods. For example, if a fully diversified portfolio in the first period had continued to hold 5 stocks in the second bull market condition, it would have lost around 50% of the diversification benefit at that period. Fully diversified bear portfolios also showed the same result. A fully diversified portfolio in the first bear period lost around 30% of the portfolio diversification benefit in the second period. However, the study signifies that a differently weighted portfolio should hold more number of stocks in the bear period than that in the bull period to get full diversification benefit.

However, naive portfolio analysis showed consistency in the number of stocks naive portfolios should hold to get full diversification benefit in both bull and bear market conditions. As illustrated in table 3, a fully diversified portfolio held 12 stocks in the overall period under study. A fully diversified portfolio held 13 stocks and 10 stocks in the first and second bear period respectively, whereas a fully diversified portfolio held 15 stocks and 9 stocks in the first and second bull period respectively. Moreover, the fluctuation of diversification benefits in different market conditions was minimal. Diversification benefit fluctuated around 10% in all market conditions. Portfolio volatility around 10% should be tolerable. The author concluded that fully diversified naive portfolios should hold 9 to 15 stocks. The findings say that naive portfolios in the bear period do not necessarily hold more stocks than those in the bull period.

5. Conclusions

Unsystematic risk has no compensation in the marketplace. However, it is not always possible to hold all the stocks in a portfolio to wipe out unsystematic risk. Stock liquidity, stock breadth, and transaction costs among others affect portfolio managers' ability to fully replicate the benchmark index. Accordingly, portfolio managers take position in some selected securities and try to get as much diversification benefit as possible. So it is important to determine the optimum number of stocks to be included in portfolios to get full diversification benefit.

The earlier research conducted by Reilly (1985) found that the number of stocks in a portfolio should be 12 to 18 to eliminate full non-systematic risk. Newbound and Poon (1993) found that a portfolio should have more than 20 stocks to get full diversification benefit. Statment (1987) found that a diversified portfolio should have at least 30 randomly selected stocks for borrowing investors and at least 40 stocks for lending investors. The most recent research by Vilija Alekneviciene', 2012 showed that a naive portfolio and a differently weighted portfolio should have 24 stocks and 27 stocks respectively to get full diversification benefit. Our study suggests that we cannot definitely identify the range of the number of stocks differently weighted portfolios should hold to get full diversification benefit. But naive portfolios should hold 9 to 15 stocks to get full diversification benefit.

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	Diversification Effect		
	Differently		
	Weighted		
	Portfolio	Naive Portfolio	
Portfolio 1	72.08	66.07	
Portfolio 2	100.00	91.53	
Portfolio 3	84.56	76.93	
Portfolio 4	89.22	82.52	
Portfolio 5	51.38	78.16	
Portfolio 6	59.03	85.81	
Portfolio 7	64.17	88.58	
Portfolio 8	68.51	92.91	
Portfolio 9	69.91	93.69	
Portfolio 10	75.86	97.57	
Portfolio 11	80.06	100.00	
Portfolio 12	81.02	98.48	
Portfolio 13	81.46	96.09	
Portfolio 14	84.37	97.05	
Portfolio 15	84.66	96.36	
Portfolio 16	85.35	96.94	
Portfolio 17	85.96	97.23	
Portfolio 18	85.36	94.27	
Portfolio 19	85.54	94.48	
Portfolio 20	84.67	90.46	
Portfolio 21	83.43	89.41	
Portfolio 22	81.56	86.41	
Portfolio 23	81.23	83.90	
Portfolio 24	79.99	81.85	
Portfolio 25	80.10	81.50	
Portfolio 26	79.07	80.20	
Portfolio 27	78.59	79.32	
Portfolio 28	78.28	78.01	

Table 1: Diversification effect (DE) ratio

	Overall Portfolio	Bear Portfolio 1	Bear Portfolio 2	Bull Portfolio 1	Bull Portfolio 2
Portfolio 1	72.08	19.61	56.78	79.67	57.07
Portfolio 2	100.00	34.29	60.76	84.28	85.04
Portfolio 3	84.56	48.44	82.75	95.05	100.00
Portfolio 4	89.22	57.73	81.42	100.00	53.82
Portfolio 5	51.38	66.00	92.48	49.78	60.38
Portfolio 6	59.03	69.10	95.49	58.17	72.70
Portfolio 7	64.17	77.13	96.55	62.87	74.83
Portfolio 8	68.51	77.83	98.40	64.90	84.19
Portfolio 9	69.91	78.85	100.00	69.46	81.98
Portfolio 10	75.86	84.59	61.52	71.09	87.06
Portfolio 11	80.06	91.32	65.89	75.84	89.41
Portfolio 12	81.02	96.53	68.71	79.34	89.46
Portfolio 13	81.46	96.78	69.22	80.34	91.70
Portfolio 14	84.37	100.00	68.45	57.11	92.00
Portfolio 15	84.66	93.22	68.46	30.78	94.34
Portfolio 16	85.35	97.98	69.20	84.98	92.39
Portfolio 17	85.96	98.46	69.25	86.41	92.67
Portfolio 18	85.36	96.90	66.71	86.50	90.75
Portfolio 19	85.54	97.23	66.59	87.52	93.39
Portfolio 20	84.67	94.65	66.11	88.51	92.57
Portfolio 21	83.43	90.24	66.28	87.76	92.05
Portfolio 22	81.56	86.20	66.32	87.60	90.82
Portfolio 23	81.23	80.58	66.33	87.68	86.38
Portfolio 24	79.99	80.38	66.45	87.76	84.58
Portfolio 25	80.10	80.27	67.18	87.85	84.23
Portfolio 26	79.07	76.52	66.60	88.16	83.70
Portfolio 27	78.59	76.02	65.16	87.85	78.75
Portfolio 28	78.28	75.55	65.10	87.80	77.84

Table 2: Diversification effect (DE) ratio of differently weighted portfolio in different market conditions

	Overall Portfolio	Bear Portfolio 1	Bear Portfolio 2	Bull Portfolio 1	Bull Portfolio 2
Portfolio 1	66.07	49.42	78.59	75.81	49.07
Portfolio 2	91.53	75.37	80.28	74.87	73.31
Portfolio 3	76.93	93.50	95.04	88.38	92.08
Portfolio 4	82.52	90.58	83.09	95.74	89.56
Portfolio 5	78.16	87.09	90.60	85.33	90.78
Portfolio 6	85.81	92.56	94.61	90.90	98.92
Portfolio 7	88.58	94.35	95.42	91.43	96.86
Portfolio 8	92.91	93.04	98.36	92.49	100.00
Portfolio 9	93.69	93.45	100.00	95.42	85.64
Portfolio 10	97.57	96.44	89.59	97.06	88.69
Portfolio 11	100.00	98.60	91.82	98.31	91.02
Portfolio 12	98.48	100.00	93.21	98.33	90.09
Portfolio 13	96.09	96.25	91.52	98.57	92.06
Portfolio 14	97.05	96.10	87.37	100.00	91.87
Portfolio 15	96.36	84.71	86.04	99.17	92.67
Portfolio 16	96.94	85.95	84.60	99.34	91.51
Portfolio 17	97.23	84.77	84.67	99.42	91.86
Portfolio 18	94.27	82.53	82.24	98.15	88.93
Portfolio 19	94.48	82.14	78.87	97.58	90.21
Portfolio 20	90.46	79.62	76.88	97.85	88.04
Portfolio 21	89.41	75.77	76.49	95.11	87.62
Portfolio 22	86.41	72.75	76.31	94.11	82.84
Portfolio 23	83.90	67.90	75.32	94.11	79.33
Portfolio 24	81.85	66.72	74.86	93.62	77.58
Portfolio 25	81.50	66.18	75.16	92.94	76.69
Portfolio 26	80.20	64.60	73.83	92.97	75.64
Portfolio 27	79.32	63.46	71.90	91.94	72.52
Portfolio 28	78.01	62.26	71.74	91.33	70.02

Table 3: Diversification effect (DE) ratio of naive portfolio in different market conditions

Appendix 1: Companies included in the study are as follows:

Company Name	Symbol
Apple Inc.	AAPL
American Express Company	AXP
The Boeing Company	BA
Caterpillar Inc.	CAT
Cisco Systems, Inc.	CSCO
Chevron Corporation	CVX
E. I. du Pont de Nemours and Company	DD
The Walt Disney Company	DIS
General Electric Company	GE
The Goldman Sachs Group, Inc.	GS
The Home Depot, Inc.	HD
International Business Machines Corporation	IBM
Intel Corporation	INTC
Johnson & Johnson	JNJ
JPMorgan Chase & Co.	JPM
The Coca-Cola Company	KO
McDonald's Corp.	MCD
3M Company	MMM
Merck & Co. Inc.	MRK
Microsoft Corporation	MSFT
Nike, Inc.	NKE
Pfizer Inc.	PFE
The Procter & Gamble Company	PG
The Travelers Companies, Inc.	TRV
UnitedHealth Group Incorporated	UNH
United Technologies Corporation	UTX
Verizon Communications Inc.	VZ
Wal-Mart Stores Inc.	WMT
Exxon Mobil Corporation	XOM

Table A: Companies included in the analysis

Source: Yahoo Finance