

## An Educational Experiment for Facilities Location Problems

**Cem Canel**

University of North Carolina Wilmington  
Cameron School of Business  
Department of Information Systems and Operations Management  
601 S. College Road, Wilmington, NC 28403-5611  
United States of America

**Sidhartha R. Das**

George Mason University  
MS# 5F4MS# 5F4 School of Management  
4400 University Drive, Fairfax, VA 22030-4444  
United States of America

### Abstract

---

*This paper describes the development of an educational experiment for facilities location problems. The purpose of this experiment is two-fold: first is to provide students with a better understanding of various issues involved in location decisions, and second, to expose them to a real facilities location problem in the service sector - specifically for an insurance company. To achieve this goal we conducted an experiment with students enrolled in an introductory operations management course. The educational experiment developed as a consequence of this experiment will help enhance student knowledge about location decisions.*

---

### Introduction

The proliferation of internet based distribution channels for products and services and greater competition in both national and international markets have caused businesses to search for potential areas of savings while offering better customer service. In the last two decades, major changes in business operations have raised important questions that are related to the location of facilities (e.g., offices or warehouses) in a company's distribution system. For example, Amazon's recent decision to deliver products ordered by customers within a few hours (in selected markets) requires a distribution system of warehouses that are strategically located to fulfill customer demand at a minimum cost. Students in operations management courses need to be aware of the variety of methodologies used in solving these types of problems. However, one of the tasks faced in teaching operations management to students is the illustration of the applicability and relevance of operations management techniques to business problems. The utility of these techniques can be conveyed to students by providing them with business problems and the application of appropriate methodologies to solve these problems.

The distribution of products or services is an important area that offers a high potential for savings. It includes determining the location of new facilities, which is an important managerial issue due to increases in transportation costs, and changes in demand patterns. The term "facilities" can include the following: distribution centers, manufacturing plants, offices, medical clinics, fire stations, post offices, libraries, refineries, schools, hotels, etc.

Facilities location problems typically involve the following parameters [3,4]: the cost of distribution to and from the facility, volume of distribution, fixed and variable facility costs, service performance of the facility and potential increases in demand. An open facility incurs a fixed cost and a variable operating cost which is a function of its throughput. In addition to these costs, transportation costs to and from the facilities are involved. The problem is then to select the optimal locations where facilities should be opened. These selected locations must supply the demand coming from the customers without exceeding the capacity constraints of open facilities and minimize the sum of the fixed, variable and transportation costs.

If the variable and transportation costs are assumed to be linear, the facility location problem can be formulated as a zero-one mixed integer linear programming problem. In the case where these costs are not linear, zero-one non-linear programming techniques can be used.

The general mathematical formulation of the facility location problem (FLP) is given as follows [3]:

$$\text{MIN } \sum_{i=1}^N \sum_{j=1}^M C_{ij} X_{ij} + \sum_{i=1}^N F_i Y_i \quad 1 \quad (1)$$

Subject to:

$$\sum_{i \in N} X_{ij} \geq D_j \quad j \in M \quad (2)$$

$$\sum_{j \in M} X_{ij} \leq S_i Y_i \quad i \in N \quad (3)$$

$$X_{ij} \geq 0 \quad (4)$$

$$Y_i = \begin{cases} 1 & \text{if facility } i \text{ is opened} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Notations used in the formulation

N	Number of potential facilities ( <i>i</i> ).
M	Number of customers ( <i>j</i> ).
$F_i$	The fixed cost associated with facility <i>i</i> .
$C_{ij}$	The variable cost of shipment from facility <i>i</i> to customer <i>j</i> .
$X_{ij}$	The quantity shipped from facility <i>i</i> to customer <i>j</i> .
$D_j$	Demand from customer <i>j</i> .
$S_i$	Capacity of facility <i>i</i> .

*Fixed cost ( $F_i$ ):* This cost is associated with facility *i*. Fixed costs at the potential facilities are composed of training costs, personnel costs, and other administrative costs.

*Variable Costs ( $C_{ij}$ ):* The variable costs include labor costs, material costs, utility cost, and other per unit costs.

*Demand ( $D_j$ ):* This denotes demand in market *j*.

*Capacity ( $S_i$ ):* This denotes capacity of facility *I*.

The objective function is the minimization of the total cost formed by the sum of transportation costs, facility operating costs, and total fixed costs. Constraint set 2 requires that each customer's demand is satisfied and constraint set 3 restricts the total shipments from site *i* to its capacity,  $S_i$ , only if the facility is open ( $Y_i = 1$ ). No shipments are permitted when it is closed ( $Y_i = 0$ ).

These selected facilities must supply all demand at a predetermined set of customer territories, and should minimize the sum of the fixed, variable and transportation costs. In the case where each facility has a limit on its capacity, the FLP is termed as the capacitated FLP [3]. On the other hand, when each facility has the capacity to satisfy all the customers' demand, the problem is called uncapacitated FLP [4].

Most of the text-books in operations management [2, 5, 7] cover mainly three primary methods for solving facilities location problems. The first method is the factor rating method. The second method is the center of gravity method, and the third one is the transportation approach. These three methods address some of the underlying issues in locating facilities, but for most of the students, they are merely number crunching without requiring them to think any further.

This study discusses an educational experiment for explaining facilities location problems, their importance, the various issues involved, and the solution methods in a realistic business context. In an application paper, Gelb and Khumawala [1] applied FLP solution methods to determine the office reconfiguration of the Variable Annuity Life Insurance Company (VALIC), a Houston-based insurance marketing firm offering products to employees of non-profit and government organizations at all levels.

As stated in the paper, VALIC was interested in organizing its salesforce of 336 individuals in order to serve 57 geographic segments - states or portions of states - and to reduce its total costs [1]. They assumed that the largest cities in these 57 geographic segments were the potential locations where offices might be established. Thus, the problem consisted of 57 potential locations and 57 customers (geographic segments).

We capitalized on the VALIC study to develop an educational experiment to illustrate the importance of FLP and to provide the students with a better understanding of FLP. In the next section, we provide some background on the actual location problem faced by VALIC and the experiment conducted in this study.

### Design of the Experiment

The purpose of their project was to divide the United States into sales territories which would be serviced at the minimum cost level under certain constraints. The study showed that some smaller territories had fixed costs that were unnecessarily high and that some larger territories had salespeople who were spending excessive amounts of travel time, which could otherwise be spent in sales efforts. VALIC's problem was to organize and locate their sales force of 336 personnel so as to obtain an optimal cost subject to certain constraints. At the time of this study, VALIC had 16 territories which had evolved as combinations of 57 segments. The factors that VALIC wanted to determine were: the costs associated with their existing 16 territories, the optimal number and locations of territories to serve their markets, and the amount of savings in a new territorial configuration. In order to ensure that the total number of sales persons in any given territory would not change, it was assumed that the VALIC sales force would be assigned to a particular region in a proportion equal to the portion of US market potential in that region.

The organizational problem cited by VALIC indicated that a trade-off would be necessary between fixed and variable costs that are usually related to combinatorial problems. Such problems are amenable to solution by using implicit enumeration techniques like branch and bound algorithms and specially designed heuristic algorithms.

The solution technique used for solving VALIC's problem was a modified version of the branch and bound algorithm developed by Khumawala [3]. It was employed in this project to determine the number of locations that would be optimal for VALIC, where the territories should have their offices, and which geographic market segments should be served from each office. VALIC did not want to limit any region to a particular size, as long as the ratio of largest and smallest market was within a reasonable limit. Thus, capacity constraints were eliminated from the model, and the resulting problem was solved as an uncapacitated facility location problem.

With the assumptions mentioned above and VALIC would continue to employ 336 salespersons nationwide, costs fell from \$18,825,967 to \$9,993,622 (almost 50%) for the recommended configuration plus a one-time cost for new offices.

A subset of the VALIC problem presented to Gelb and Khumawala [1] was chosen as the FLP for this experiment. In this study, we selected 23 of the original 57 geographic segments (customers) located in the Northeast corner of the United States and 18 of the largest cities (potential locations) in these 23 geographic segments. By selecting 23 geographic segments and 18 largest cities, we attempted to make the problem somewhat "challenging" for the students and at the same time not get them "bogged down" in mere number crunching.

The students included in this study were enrolled in three sections of an undergraduate operations management elective. At the time the experiment took place the students had no prior exposure to the FLP. The students in this study were provided with

1. A map of Northeastern United States showing the 23 geographic segments and 18 largest cities; the geographic segments were numbered 1 through 23 and the largest cities were represented by letters *a* through *r* (Exhibit I),
2. A separate form presenting the names of the geographic segments and the largest cities, (Exhibit II), and
3. The cost matrix displaying the cost of satisfying the demand in the geographic segments from the largest cities to these geographic segments and the fixed costs for each potential location (Exhibit III).

Participating students were also provided with one page of instructions, explaining that they were assigned the task of selecting the location of the offices to serve the customers in the 23 geographic segments and determine which geographic segments should be served from the selected offices. The students were also told that their objective should be to minimize the total cost such that the demand from all of the customers should be satisfied and that each customer should be served only from one office.

We obtained the optimal solution to this reduced problem by formulating the problem as a mixed integer programming problem and then solving it with LINDO [6]. The value of the objective function for the optimum solution to the problem, without the configuration of the offices, was given to the students in the beginning so that they would be able to compare their solutions to the optimal solution and not just “shoot in the dark.”

As a pilot study, 12 students taking an elective course in operations management were elected to solve the problem manually. These students had already been exposed to FLP in their prior introductory operations management course. The purpose of the pilot study was to get feedback about the experiment before applying it in other classes. These students were asked to form groups of two, thus resulting in 6 groups. The results of the pilot study were helpful in several ways; for instance having the costs in the cost matrix clustered around the diagonal made it easier for them to solve the problem. Thus, this feedback was incorporated, and the structure of the cost matrix was modified. It also helped in determining the average time required to solve this problem. With this modification and the helpful feedback on timing, etc. we employed the experiment in the other three classes.

At the beginning of each class, we spent approximately 25 minutes explaining the experiment to the students and answering their questions. Some of the issues asked by the students were;

*Who were the customers?*

*How were the customers selected?*

*Could the clients be served from more than one office?*

*Why were the fixed costs of all these offices assumed to be equal?*

*How were the costs determined?*

*What was the planning horizon for these location decisions?*

*Was minimization of total costs the only criterion in determining the locations?*

Clearly, these kinds of questions raised by the students, and the ensuing discussion among them was most helpful in their understanding of the various issues involved in determining the “best” locations for facilities. Also, we ensured that they understood the assumptions VALIC had made, as we presented them the reduced problem and data.

The students were asked to form groups of two and were given a maximum of 35 minutes to provide their best solution. In total, 104 students were involved. They were also told that they would have two chances to improve the first solution they obtained within the given time frame. They were assisted in calculating the total costs for their configurations. An Excel spreadsheet for calculating the total costs was developed and used to

1. Verify all of the total costs provided by the students,
2. Make sure that no geographic segment was supplied from more than one office, and
3. Check that demand from all of the geographic segments was satisfied.

## **Results and Conclusions**

The total cost obtained for the optimum solution to this problem was \$4,137(000). Some groups calculated the total cost without any assistance. Fifteen of the 34 teams with the solution of 4,137 reported that they obtained this answer in their second attempt. All of the groups with a solution of maximum 7.64% higher than optimum solution stated that they were “satisfied” with their solution and did not want to attempt to improve their answer even when they had time. The groups whose solutions were at least 19.36% higher than the optimum solution either did not have the time to improve their answer or did not want to do so.

The following points are noteworthy: The results obtained by the students indicate that all of the solutions were feasible in terms of satisfying the constraints and following the instructions given to the students.

The FLP problem in this experiment had multiple optimal solutions, and this may substantiate why such a large percentage (65.4%) of the groups got the optimum solution. If we had selected a problem with a unique optimal solution, the results perhaps might not have been the same. We, of course, had taken the cost data as they were in the VALIC study of Gelb and Khumawala [1], In order to have only one optimum solution to the problem we would have had to modify it. As stated earlier, our objective here was to develop an “experiment” for better education and not to compare how students would solve FLP versus the computer’s optimum solution. A possible by-product or motivation from this experiment would be to do research in this line.

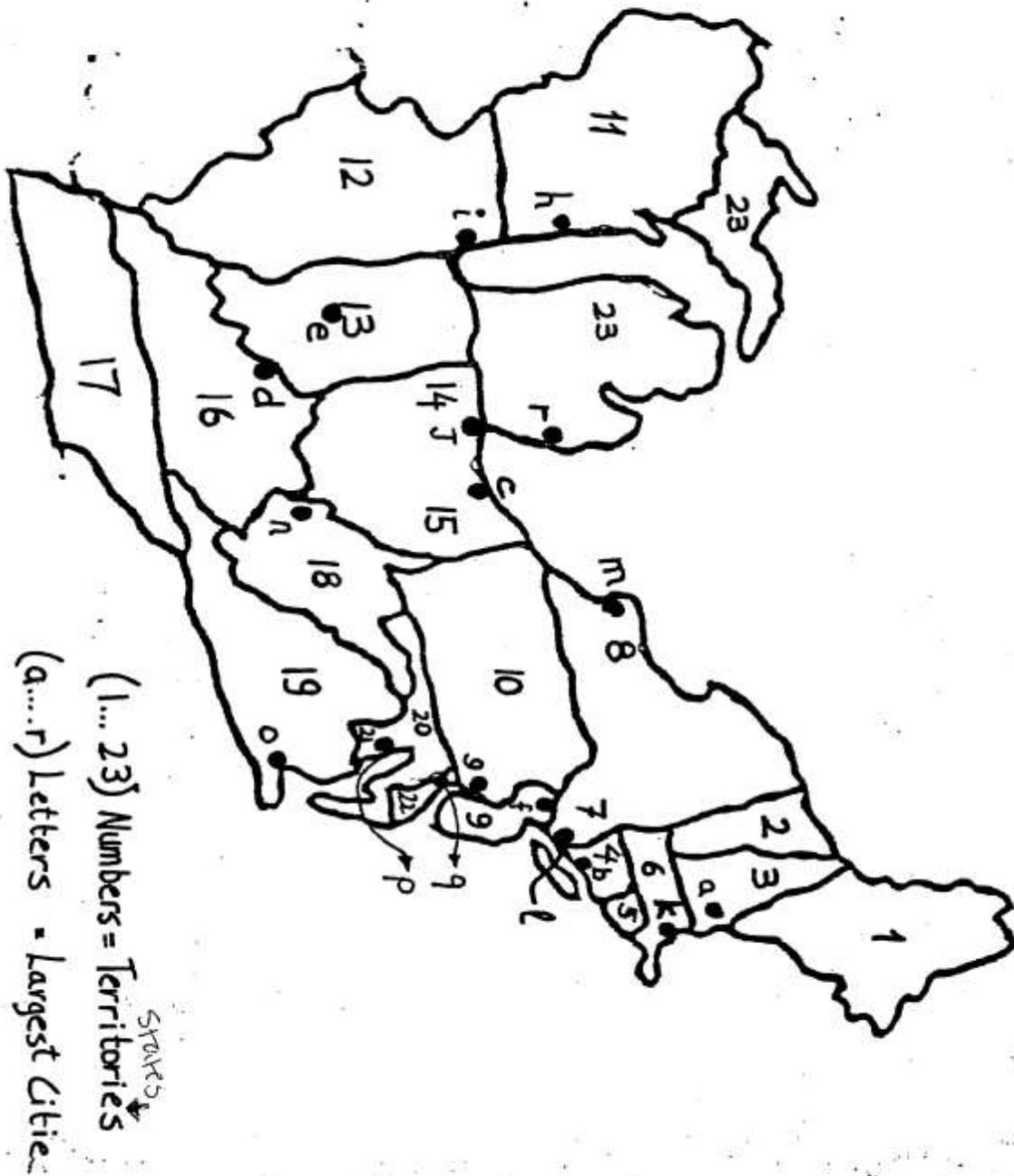
The aforementioned questions asked by the students and the resulting discussion would probably not have been achieved if they were not given the opportunity of using this educational experiment. Of course, we dealt with these questions in subsequent classes.

It would be of interest to inquire from the students who participated in developing this experiment as to how it helped them in understanding the FLP, vis-a-vis the typical lecture/textbook/hypothetical small example format. The results would help us determine if such experiments were beneficial in other areas of operations management as well. The educational experiment developed in this paper will be helpful in demonstrating the importance and relevance of the techniques developed for solving the FLP.

### **References**

- [1] Gelb, B. and B.M. Khumawala, (1984). Reconfiguration of an Insurance Company's Sales Regions. *Interfaces*, 14, 87-94
- [2] Heizer, Jay, B. Render, and C. Munson, (2017). *Principles of Operations Management*. Pearson.
- [3] Khumawala, B.M., (1972). An Efficient Branch and Bound Algorithm for the Warehouse Location Problem. *Management Science*, 18 (12), 718-731.
- [4] Khumawala, B.M., (1973). An Efficient Heuristic Procedure for the Uncapacitated Warehouse Location Problem. *Naval Research Logistics Quarterly*, 20 (1), 109-121
- [5] Krajewski, Lee J., M. Malhotra, and L. P. Ritzman, (2016). *Operations Management*. Pearson.
- [6] Linus E. Schrage, (1986), *Linear, Integer, and Quadratic Programming with Lindo*. Scientific Press.
- [7] Stevenson, William J. (2015). *Operations Management*. McGraw-Hill.

EXHIBIT I



**EXHIBIT II**

	Territories	Largest City	Will serve territory #
1	Maine		
2	Vermont		
3	N. Hampshire	a - Manchester	
4	Connecticut	b - Bridgeport	
5	Rhode Island		
6	Massachusetts	k - Boston	
7	S.E. New York	l - New York City	
8	N. New York	m - Buffalo	
9	New Jersey	f - Newark	
10	Pennsylvania	g - Philadelphia	
11	Wisconsin	h - Milwaukee	
12	Illinois	i - Chicago	
13	Indiana	e - Indianapolis	
14	N.W. Ohio	j - Toledo	
15	S.E. Ohio	c - Cleveland	
16	Kentucky	d - Louisville	
17	Tennessee		
18	W. Virginia	n - Huntington	
19	Virginia	o - Norfolk	
20	Maryland		
21	Washington, D.C.	p - Washington, D.C.	
22	Delaware	g - Wilmington	
23	Michigan	r - Detroit	

**EXHIBIT III**

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Fixed Cost
a	14	9 1	12	49 4	1 4	90																		152
b	17 5	9 1	15 5	40	1 4	11 28	14 8		11 99	19 18														152
c										19 18			76 2	7 0	12 7	50 6		27 4	99 6				13 80	152
d													61	7 0	15 83	40	74 2	27 4					13 80	152
e											70 5	18 11	6 1	70	15 83	50 6	74 2	27 4					13 80	152
f				40			18 51		96	19 18												8		152
g				49 4				16 57	11 99	15 3					15 83			27 4	99 6	90 4	66 3	10 3		152
h											5 6	14 5	76 2											152
i											5 6	14 5	76 2	7 0									13 80	152
j												18 11	76 2	6	15 83	50 6		27 4					11 0	152
k	14	9 1	12	49 4	1 4	90		16 57																152
l				40			14 8		11 99															152
m		9 1				11 28		13 3		19 18													10 3	152
n										19 18				7 0	15 83	50 6	74 2	22	99 6	90 4				152
o										19 18					15 83			27 4	80	90 4	66 3	10 3		152
p										19 18									99 6	72	53	8		152
q								16 57	11 99	19 18								27 4		72	53	8		152
r										19 18		18 11	76 2	6	15 83	50 6							11 0	152