

Teaching Differential Equation with Technology: The Improved Euler's Method

Dr. Michael Gallis

Dr. Ping Wang

Penn State University, Schuylkill
United States of America

Abstract

In Differential Equation, the Improved Euler's Method is an effective numerical method used for solving initial value problem. The formula is complicated and confusing. In this work, we will use computer programs to generate graphics/animation to visually demonstrate the process and ideas involved, hence improving students' understanding and learning of the approach.

Introduction

It is well known that many students across the country are intimidated by mathematics and therefore are doing very poorly in mathematics. One of the reasons is that the abstraction, the logic thinking and the complex jargon that are involved in math make it very difficult for students to understand the basic concepts. Many students struggle during the traditional (lecture and text based) process of learning math because they do not have a clear mental picture of what is going on. Graphics and interactive computer programs can be very useful in introducing the mathematical concepts visually when they are discussed in class because students have the relevant pictures and processes in mind.

There is a rich collection of literatures on how technologies can improve mathematics learning such as Herold [1], Kersaint [4], Lee [6], Mishra [7] and Schrum [8]. We have been exploring various computer software, programs that can be integrated into mathematics education to improve student learning. Different technology can be used in different subjects to maximize their usefulness. For example, matlab, one of the earlier programs we utilized is a very powerful tool for computations in matrix analysis. We used matlab to demonstrate solutions of partial differential equations (Other application of Matlab in advanced mathematics can also be found in Yang & Ma [9]). In recent years, we have also experimented with Aleks, an online learning system, which allows instructors to assess students' readiness for further mathematics courses. Other technology tools that were utilized in our efforts to enhance mathematics education include java script, POV-Ray, Photoshop, Mathematica, etc.

Improved Euler's Method

This is a part of a pedagogical research project. Our goal in this paper is to examine the Improved Euler's Method in differential equations. The mathematical formula presented in textbook such as Zill [10] is complicated and difficult to understand. The result and mathematical idea involved can be better demonstrated and understood via the use of technology, with the creation of animated graphics for relevant expressions and equations.

Consider an initial problem

$$y' = f(x, y) \quad (1)$$

$$y(0) = y_0 \quad (2)$$

When it is impossible to find a solution of (1), (2) in closed form, we try to obtain numerical solution of the problem. In Differential Equation class, an important method discussed is called Improved Euler's Method. With a step h , a predetermined small fixed increment of x , the formula for this method is a successive process

$$y_{n+1} = y_n + h \frac{f(x_n, y_n) + f(x_{n+1}, y_{n+1}^*)}{2} \quad (3)$$

$$y_{n+1}^* = y_n + h f(x_n, y_n) \quad (4)$$

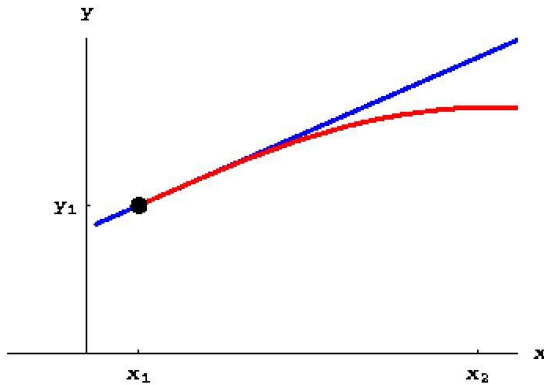
This formula is complicated and difficult to understand. Students can follow the instruction to apply the formula but hardly knowing what is really going on with the formula itself.

Based on the principle involved, using software Mathematica, we are able to generate a sequence of graphics to create an animation (cf. Gallis, M. & Wang, P. [2], [3]) to illustrate the idea of the method and the interpretation and implication of terms and expressions in the traditional formula, helping students really understand the topic.

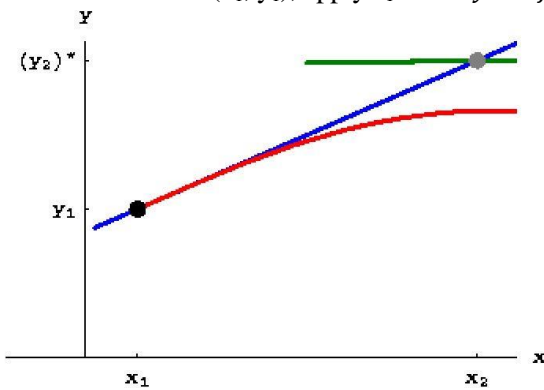
Graphic Illustration

In this section we extracted a sequence of graphics from the animation to explain the mathematical idea and method involved in Euler’s improved method through the following Step 1 to Step 4:

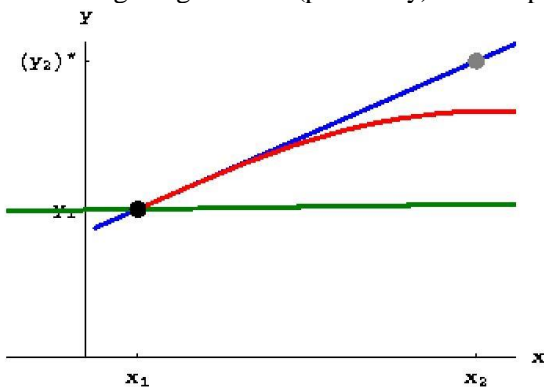
1. Starting at (x_1, y_1) , apply $k_1 = y' = f(x_1, y_1)$ as slope to generate a straight line in blue.



2. Along the blue line, move a step h , arriving at (x_2, y_2^*) . Note that y_2^* is the original Euler’s approximation solution. At (x_2, y_2^*) , apply $k_2^* = y' = f(x_2, y_2^*)$ as slope to generate a straight line in green.



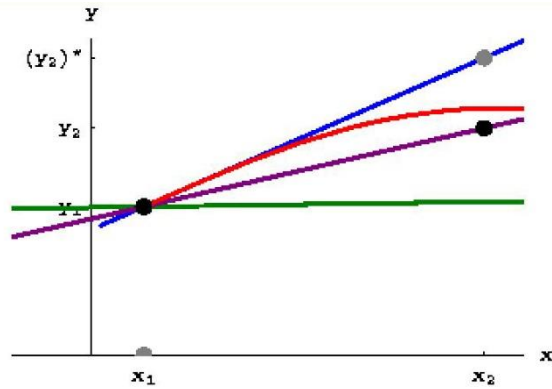
3. Bring the green line (parallel-ly) back to point (x_1, y_1) .



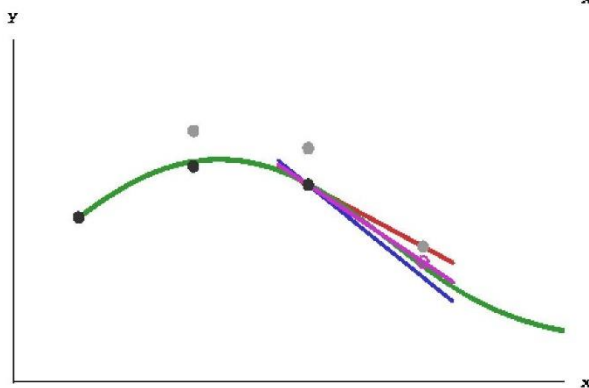
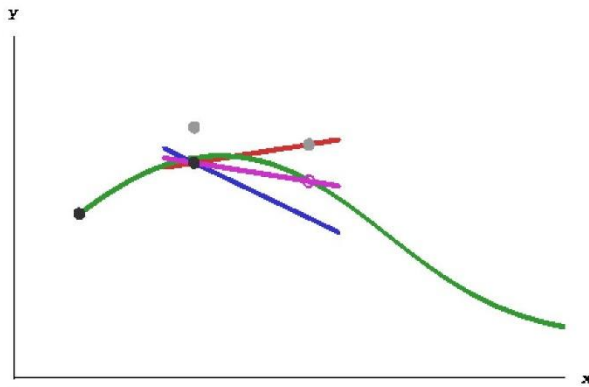
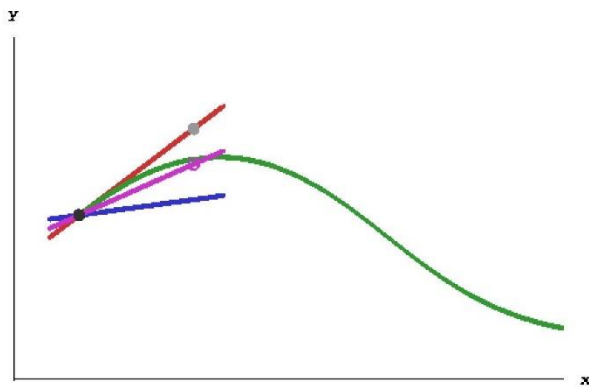
4. Draw a purple line in the middle of the blue line and green line, mathematically with a slope of

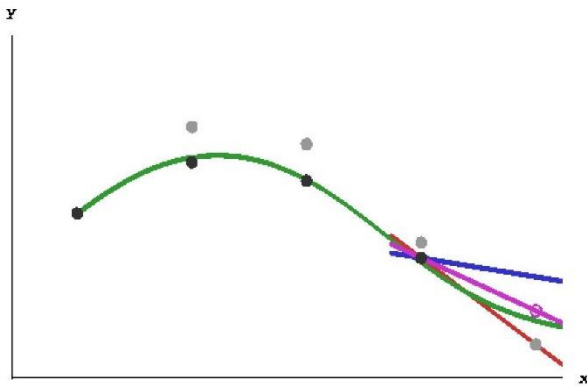
$$k_2 = \frac{k_1 + k_1^*}{2}$$

Then move along the purple line one step h , arriving at (x_2, y_2) . This value of y_2 now is the successive approximate value of the solution of initial value problem (1), (2), according to the Improved Euler's Method.



The next sequence of photos illustrates the successive approximation process, repeating Step 1 through Step 4, moving at a step of h at a time:





This sequence of images are not just photographic illustration of the idea of the Improved Euler's Method. They are in fact based on an actual initial value problem. While the green curve representing the actual solution, the black dots are the approximate solutions of the initial value problem (1), (2).

Needless to say the Improved Euler's Method produce much more accurate approximate solutions as demonstrate numerically in any differential equation textbook, such as Zill [10]. Our goal is not to address the accuracy of the method, rather to illustrate, through graphics/animation, the principle involved in the method, which is nowhere obvious to students what is going on when they look at formula (3) and (4). It is our hope, also as demonstrated through numerous literatures (cf. Mishra & Koehler [7], Yang & Ma [9]), that with knowing the process and inner work of the method, students will better understand the mathematical theory and results, not just to know how to apply the formula.

Future Plan

Throughout the entire course of Differential Equations, the teaching of many other topics, including direction field, stability/instability of solutions, numerical solutions, etc., could be enhanced by use of many other appropriate technology, some with statics graphics, some animations, while others with interactive java applets. We are planning on approaching each different topics with different technology based on the nature of the topics, with the same goal of enhancing students' learning in Differential Equation.

References

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