Assessing Mathematical Representations in Kindergarten

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Abstract

The purpose of this study was to advance understanding of assessment methods regarding kindergarten students' representations of mathematical ideas and knowledge. This investigation explored the use of children's oral language, drawings, and writing along with the use of gestures when communicating mathematically with their teacher and peers in a classroom and how a classroom teacher assessed the students' mathematical understanding.

Keywords: Assessment, mathematics, representations, kindergarten

1. Introduction

Being able to represent and interpret mathematical ideas and concepts effectively is imperative in order for children to become mathematically literate. Flevares and Perry (2001) assert that "each individual, from elementary school students to professors of mathematics, relies on symbols to represent abstract quantities and operations when communicating about mathematical ideas" (p.330). Many studies have researched and described the ways representations are used in mathematics classrooms (Goldin & Sheteingold, 2001; Kamii, Kirkland & Lewis, 2001; Whitin & Whitin, 2001; Woleck, 2001). According to the Standards produced by the National Council of Teachers of Mathematics (NCTM), representation is regarded as an important topic of the mathematics curriculum and an essential aspect for mathematical competence (2000). Woleck (2001) has articulated that "representations allow mathematicians to record, reflect on, and later recall their process and thinking. They become tools to turn to as mathematician articulate, clarify, justify, and communicate their reasoning to others" (p.215). Representation is not a method or strategy but is a part of process or means of constructing mathematical ideas and knowledge (Flevares & Perry, 2001; Kamii et al., 2001; NCTM, 2000; Whitin & Whitin, 2001; Woleck, 2001).

In order to become mathematically literate and to be able to function in today's society as a productive citizen, children must be able to represent and interpret mathematical ideas and concepts. In this changing world, children will be challenged to keep current with new technologies, to think mathematically, and to problem solve on a daily basis. Much research has been conducted on the functions of representation for older children in the past. Studies have indicated the importance on focusing on children's thought process rather than end products (Goldin & Sheteingold, 2001; Kamii et al., 2001; Whitin & Whitin, 2001; Woleck, 2001). Even though primary grade teachers realize the importance of representation in mathematics curriculum, there seems to be a gap between the current practice in classrooms and teachers' knowledge and ability to teach and assess contents articulated in the Standards sucha as the Common Core State Standards (CCSS). If we are to teach all students to become competent in mathematics, teachers of young children need to become aware of the ways in which their students represent and communicate their mathematical understanding and how to assess them so that they can guide and facilitate students learning in their classroom. The following research question guided this inquiry: How can a kindergarten teacher assess children's mathematical knowledge and ability?

2. Materials and Methods

The participants in this study were 18 predominantly middle-class kindergarten children who attended a public elementary school. This was a convenience sample. There were approximately 800 students, of which 40% qualified for the free or reduced lunch program, in a rural town in a southeastern state. The school served 80% Causasian, 18% African American and 2% Hispanic and Asian students.

2.1 Data Collection

Data were collected for this study in four ways in order to enhance the validity of the research findings. Data collection methods included observation/audiotaping, informal interviews, children's journals/conferences, and field notes during the three-week period. The primary investigator was a public school teacher who conducted this study in her kindergarten class.

Observation/Audiotaping

Observation/audiotaping were conducted throughout the normal school day in which students engaged in a variety of mathematics activities individually, in small groups, and in a whole class setting. Individual and small group mathematics activities included the writer's workshop, center time, and some of the math lessons. The primary investigator used audiotapes to record interactions between students and her to be transcribed later for analysis. The teacher kept a journal of field notes to record any mathematics-related interactions between students and her observed during the day.

Students were divided into four heterogeneous groups of four to five children. Each group stayed in a particular center for the length of 20-30 minutes during the literacy center time where they collaborated and worked together as a team or, they might choose to work individually.

Informal Interviews

All interviews were informal. Data was collected during the writer's workshop, in which students made an entry of their choice in their journal and during center time, where students participated in different learning activities with minimal adult assistance. Students engaged in the writer's workshop 10-20 minutes daily and spent 20-30 minutes in centers daily. Informal interviews followed interactions between students and the teacher to clarify a child's mathematical thinking.

The contextual nature of the questions were designed to gain an in-depth understanding of students' thoughts and perceptions regarding mathematics concepts such as numbers and comparison. The questions were phrased to clarify children's actions and comments in order to determine their mathematical thinking and learning process.

Children's Journals/Conferences

During the writer's workshop, students made an entry in their journal by drawing a picture with crayons and markers and wrote something about that picture by using letters and words they know. The children had been exposed to interactive and shared writing experiences with the teacher everyday where a teacher demonstrated conventional reading and writing strategies with the whole class. Each child was to use these strategies in an attempt to write a story or label the picture at their developmentally appropriate level.

After completing their journal entries, children had individual conferences with the teacher. The teacher utilized individual conferences to assess children's progress and evaluate needs and interests of each student. The writer's workshop allowed the teacher to spend time with each student in order to gain in-depth information on how children expressed their mathematics understanding using their drawing and writing.

Field Notes

After each interview and observation, the teacher's thoughts and reflections were recorded in a journal of field notes. Although much of the data included interactions between students and the teacher in above settings, field notes were not limited to those situations.

3. Results

Qualitative collection of data to answer the research question involved observation, informal interviews, collecting student samples, and field notes.

Observation/Audiotaping

The teacher assessed children's mathematical knowledge and understanding while she observed their interactions and conversations in their classroom.

For example, Kate, Abigail, and Ann sat on the floor with a box of pattern block one morning during center time and made different patterns. Before they started to create patterns, the girls sorted all the blocks. When the teacher asked them how they sorted the blocks, the girls responded that they made different groups according shapes and colors. Here, questioning was used to assess students' knowledge.

When the teacher introduced a plastic alligator that glows when put in water for their science and math project, most children used terms such as big, bigger, small, and smaller to describe its size. As they measured it each day and discussed its size and length, the teacher introduced the concepts of shorter and longer. Children started to use these terms as they observed and discussed Bridgett, the alligator, frequently as the days went on, even though they still used terms big, bigger, small, and smaller occasionally. The teacher used her observations of the children's concepts and usage to determine that the children were ready for the additional concepts of short and long.

Informal Interviews

Informal interviews allowed the teacher to assess children's mathematics knowledge and understanding in depth. For example, Hudson demonstrated his understanding of spatial sense when he commented that "there isn't enough room (to put another bed)!" while he and Beth worked together at the computer center. They were creating a bedroom by placing various furniture of their choice when he made the comment. The teacher asked him why they were not going be placing any other furniture, Hudson responded that "they didn't fit because it didn't look right." The teacher was able to assess his sense of area and space by observing interactions between Hudson and Elizaeth and listening to what Hudson had to say about his action.

One day in February, the teacher suggested that they make valentine's day cards after she read a book "Junie B. Jones and the Mushy-Gushy Valentine's Day" with the class. When the teacher asked children how many cards they would need, Savannah said that they would need 18 cards because there were 18 children in the room. Colin explained that they needed 19 cards because there were 18 children and the teacher. Then Elizabeth came up with the number 20. When the teacher asked why they needed 20 cards, Elizabeth explained that two children were absent on that particular day and that they also needed a card. As Elizabeth explained her reasoning, Hudson counted how many children were present in the room and said that there were 16 children in the room at the time. Elizabeth paused for a second and said, "Oh, 16, 17, 18. Then (we would need) 18."

Children's Journals/Conferences

Often time, the teacher assessed children's mathematical understanding and knowledge by looking at children's journal entries and discussing with them about their entries questioning them about why and how. For example, Ann drew two cars and two bicycles and some people in her family in her journal one day. She wrote the number 4 besides the cars to represent four wheels on each car and the number 2 besides the bicycles to represent two wheels on each bicycle. She also wrote the number 14 on the bottom. When the teacher asked her about her picture, she explained that both her mother and grandmother had a car and both her sister and her had a bicycle and that made it 14 wheels altogether. Then she posed for a second and told the teacher to wait and proceeded to draw something else in her journal. She added two more people and two more bicycle in her picture and explained that she had two cousins that she plays with a lot. She erased the number 14 on the bottom of her picture and wrote the number 16 instead. When the teacher asked her why she changed the number, Ann responded that she had added four more wheels in her picture so she needed to change the number on the bottom, also. Ann demonstrated her understanding about numbers, addition process, and one-to-one correspondence. Thus, the journals served as a springboard to questioning or means of assessment.

Field Notes

The teacher used her field notes to record children's mathematics skills and understanding. Later the researcher used this information to check off skills mastered by students in each student's progress monitoring sheet. For example, the teacher noticed that Dale and Alex were reading numbers on the ruler that they were using during center time to measure what they were creating with linking blocks one morning. By listening to their conversation, the teacher noted that Dale and Alex recognized numbers up to 32.

As the children picked up weekly homework bags on one Monday afternoon, some of them started to compare the numbers written on the bags. Hudson said that Abigail's number, 10, was bigger than his number which was 3. So the teacher called out all the numbers that were checked out on the sign-out sheet and asked children which was the largest number and which was the smallest number.

They agreed that number "2" was the smallest number. After Cortez concluded that Abby's number was the biggest, Emily disagreed that her number "11" was bigger. As the two talked about this, John made a comment that Garret's bag had the number 12 on so his was the biggest. Hudson agreed to John by saying that 12 came after 11, thus making 12 the largest number. When the teacher asked if they were sure, all children agreed that 12 was the largest number. The discussion provided the teacher to assess children's knowledge and understanding of the number sense.

One day after the class had been studying penguins, Garret made a comment that emperor penguins held their eggs for two years. The teacher noted the comment in her field notes with a reminder to review the fact and the concept of time with the class later. As the teacher discussed with the class about Garret's comment, it was obvious that many children were still confused about the concept of time at this point.

The field notes indicated the teacher used observations, listening, and questioning to assess children's mathematics knowledge and understanding. She then used a checklist to record her assessment.

The findings from the data indicated that the children in this kindergarten classroom expressed their mathematics understanding in a variety of ways and the teacher assessed children's mathematical knowledge and understanding by observing children's actions, conversations, and through questioning. To do this, the teacher used informal interviews, children's journals/conferences, and field notes. The assessment was a regular part of daily routine in this classroom. The teacher used information gathered from these assessments to evaluate her own teaching methods and planned for future lessons.

4. Conclusions

The kindergartners in this study were successful at communicating mathematically with their peers and with the teacher using their oral and written language. The teacher posed many questions in this classroom encouraging children to think mathematically and critically. Children had complex understanding of mathematics skills at this time of the school year. The mathematics concepts such as patterns and date/time that occurred on all 20 days of the study were incorporated in this kindergarten class during their calendar time from day one. Children were encouraged to figure out the pattern used on the calendar and the date to record in their journals by their teacher every morning.

The teacher collected information regarding children's progress and thinking processes by posing many questions and observing children. That information was utilized to assess each child in this kindergarten classroom. The alternative assessment methods used in this study provided the teacher with more insight regarding what the children knew than do traditional assessment methods of testing children, since some children might not test as well as they would in these situations for a variety of reasons.

The use of different types of questions in this classroom provided a rich mathematics learning environment for children. Children were asked to count lunch and snack money for accuracy. The teacher asked them to figure out how many books they needed to read in the morning before they went to centers. Children were also asked to keep counts of classmates in each center so that they would not exceed the limit of participants for each center. It was obvious that the teacher was aware of opportunities that would engage children in mathematics by holding them accountable in daily classroom situations. The recognition of this phenomenon raised the possibility that the years of experience and expertise of a teacher could influence the outcome of the study.

The findings of this study were consistent with those of previous studies (Kamii, Kirkland & Lewis, 2001; McClain & Cobb, 2001; Woleck, 2001) in that young children were capable of conducting mathematical procedures at their own developmental level and of expressing their mathematical understanding in a variety of methods. In order for the teachers of young children to assess children's mathematics knowledge and understanding, one has to be knowledgeable in young children's thinking and learning process. Teachers and researchers also need to be aware of the ways in which children communicate so that they can incorporate alternative assessment methods in their daily routine. By understanding how young children learn and communicate and utilizing various assessment methods, teachers will be able to fully support their mathematics development.

5. Implications for Future Studies

When thinking about implications for further research, teachers must examine young learners through a 21st century lens.

The findings of this study encompass the processes of problem solving, critical thinking, creativity, and collaboration. These are essential skills learned by studying science, technology, engineering, and mathematics-subjects known as STEM.

A *Framework for K-12 Education* provides specific detail on combining math, science, engineering practices, and emphasizes that students must not only know and understand (STEM) Science, Technology, Engineering, and Mathematics concepts, but also be able to use their knowledge to demonstrate conceptual understanding. Assessments include students using their understanding to solve real-world problems through the practices of the engineering design process.

After the release of *The Next Generation Science Standards*, classrooms are taking more of an interdisciplinary approach and emphasizing science, technology, engineering, and mathematics (STEM) in daily lessons. Just like mathematicians, scientists use sketches, diagrams, mathematical relationships, and simulations to explain relationships and scientific phenomena. Scientists also use science notebooks and journaling strategies to explain their conceptual understandings.

Future studies should explore the use of journaling across disciplines. An added component to the study should focus on evidence-based argumentation as a result of written communication.

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