Task-Based Interviews in Mathematics: Understanding Student Strategies and Representations through Problem Solving

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Abstract

Set in a small rural school in the southeastern United States, this study aimed to describe the problem solving strategies of fourth grade students as reflected in their representations and explanations of their solutions. Historically, these students did not perform well on statewide achievement tests, and classroom observations revealed that they had little experience with problem solving. Data were collected through individual task-based interviews with fourteen students. Problems posed were nonroutine problems which could be solved through several possible solution strategies. The format for the interviews encouraged students to discover their own methods and representations. This was in sharp contrast to the teacher centered skills-based instruction that was the norm in their mathematics classes. Other than lack of experience, this study revealed no significant evidence that students lacked the ability to learn mathematics or to engage in problems solving if they had opportunities to do so.

Key Words: problem solving, interview, mathematics, representation, tasks

1. Introduction

Formal summative testing is ubiquitous in today's society. Aside from classroom testing, the need to assess student progress in mathematics has led to development of state and national tests in which many thousands of grade K-12 students are tested every year in the United States alone. Results from these tests are compared nationally by state and often internationally. For example, in the United States, based on results of the National Assessment of Educational Progress (U.S. Department of Education, 2014), states are compared with respect to both their students' perceived mathematical abilities and their progress from year to year. Although these tests may provide information about trends in mathematics achievement, they actually provide little information about how to implement effective instructional strategies, about how students perceive the mathematics curriculum, or about how to raise the level of student engagement in doing and learning mathematics. Interviews with students provide one method of identifying misconceptions about mathematical content, understanding their beliefs about mathematics and about their own ability to do mathematics, and deepening their conceptual understanding.

2. Literature Review

The theoretical framework for this study is grounded in the work of Piaget (1978), who introduced the task-based interview to elicit student thinking about problems and who believed that true understanding takes place when the student makes discoveries for himself.

2.1. Learning and Doing Mathematics

Students may possess procedural knowledge at a high level and still have gaps in conceptual understanding. Although formal testing may enable a teacher to pinpoint concepts that are more difficult for students, it may not address why these problems exist. Without this knowledge, teachers do not have a basis for addressing these difficulties. Heng and Sudarshan (2013) found gaps between teachers' thinking about a mathematical situation and children's thinking. In their study, grade 1 and 2 teachers attributed mistakes to carelessness rather than to conceptual misunderstanding. They also mistook procedural accuracy for understanding.

By developing their skills in questioning and interviewing students, they began to address their own misconceptions about what it means to learn mathematics. For example, through clinical interviews, teachers recognized the repercussions of emphasis on key words in their instruction. To simplify students' work with simple addition, they often introduced key words such as "altogether." However, interviews with the students revealed that they incorrectly extended this idea to multiplication problems such as, "If there are four cookies on each plate and five plates of cookies, how many cookies are there altogether?"

Jenkins (2010) used structured interviews of middle grades students to develop prospective teachers' knowledge of how students think and reason about mathematics. These interviews were driven by problem solving tasks. Students in an undergraduate methods course first solved a mathematics problem themselves and made lists of anticipated student responses. Working in pairs with one serving as interviewer and one serving as recorder, they posed the problem to middle school students. They then analyzed findings such as problem solving strategies, communication of solutions, and representations used. An important task in this analysis was to identify misconceptions. The prospective teachers engaged in three rounds of interviews during the semester so that the third interview was informed by the previous experiences. At the end of this experience, students exhibited a heightened ability to assess and analyze students' mathematical thinking.

2.2 Affective Factors in Learning Mathematics

McDonough and Sullivan (2014) used interviews of 8 and 9 year old children to address more affective aspects of their children's views of learning mathematics. Through a series of visual, verbal, and text-based prompts, the researchers uncovered children's beliefs about such factors as helpful or harmful learning environments, self-help strategies, and effectiveness of various strategies. Even children identified as being low achievers could articulate their beliefs about the value of strategies like estimation, working with a partner, using manipulatives, and relying on teacher instructions.

Darragh (2014) investigated students' beliefs about what it is to be good at mathematics and their beliefs about themselves as mathematics students. Students in transition to secondary school, 12 and 13 years old, were asked to describe a person who is good at mathematics. Characteristics such as getting high marks on tests, finishing work quickly, and "just knowing it" were those most often described. When asked if they met their own criteria for being good at mathematics, only one of the 22 students responded positively even though many of them received high marks in mathematics and some were in advanced mathematics programs. This result has implications for the choices students might make in their future studies and the limitations they might perceive with respect to career choices.

2.3 Deepening Conceptual Understanding

Task-based interviews provide opportunities to assess student conceptual knowledge, but they may also provide opportunities to extend that understanding. In a task-based interview, the student being interviewed interacts with the interviewer within a task environment. A key component in providing this environment is a carefully chosen task (Maher & Sigley, 2014). The interview protocol may be structured with interviewer's prompts and responses planned in advance, or it may be semi-structured, allowing for the interviewer to judge the proper response to students' mathematical reasoning. In some cases, interviews may include small groups of students or problems may be open-ended. Through questioning, the interviewer may motivate students to self-correct when they make mistakes or to extend or generalize a problem. Through the interview, the students are encouraged to examine their own strategies and their own mathematical thinking, thus extending their conceptual understanding of the situation.

In a 12-year longitudinal study, Carolyn Maher and her associates at Rutgers University (Griswold, 2000) posed several such tasks to the same group of children from their early grades to graduation from high school. Most of these problems were combinatorics problems that could be solved concretely by counting or could be extended and generalized, depending on the mathematical sophistication of the problem solver. One of these tasks, the tower problem, was posed repeatedly, varying the number of cubes in the tower. "How many different towers four cubes tall can one build when choosing from cubes of two different colors?"

At the beginning of the study, students used simple counting strategies. However, in a very short time they began to look for more efficient counting methods and were able to solve more and more complex versions of the original problem. The role of the interviewer was not to affirm students correct responses but to ask students to justify their own solutions. Throughout the study, one question was always at the forefront, "How do you know?"

3. Method

This study emerged from a professional development project provided for mathematics teachers in a small K-12 school in a rural low socio-economic environment in the southeastern part of the United States. Traditionally, students in the school had not scored well on the state mandated yearly testing. The professional development project was centered on developing both procedural knowledge and conceptual understanding through problem solving, communication using multiple representations, and connections to students' everyday lives. As director of this project, I was in the school weekly for an entire academic year. I observed many mathematics lessons at a variety of different grade levels. In general, the traditional textbook assignments that were used by most teachers did not access the mathematical talent of the students or engage them in meaningful tasks. This lack of attention to deepening conceptual understanding produced a cycle in which students did not score well on tests and were, therefore, given remedial skills-based work. There was a general lack of confidence in students' abilities to engage in deep problem solving.

I determined that semi-structured task-based interviews might provide more information about what students were able to do and might even extend their confidence and their ability to solve future problems. I selected two problems based on Maher's 12-year study (Griswold, 2000).

1. Tower Problem: How many different towers three cubes high can you make by selecting from cubes of two colors?
2. Pizza Problem: How many different pizzas can you make when choosing from three different toppings: peppers, pepperoni, and sausage? For the purposes of this problem, you cannot make half pizzas like half peppers and half sausage. However, you can make a pepper and sausage pizza.

These two problems are isomorphic. Two problems are isomorphic if they are solved in the same way, even though the contexts and the names for the elements of the problems are different. The same strategies can be applied to achieve a solution and the structures of the solutions are the same. When choosing from red and blue cubes, the possible towers can be sorted into the following groups: one tower with no blue cubes, three towers with one blue cube, three towers with two blue cubes, and one tower with all blue cubes. For the pizza problem, the pizzas can be sorted into the following groups: one pizza with no toppings, three pizzas with one topping, three pizzas with two topping, and one pizza with all toppings.

Because state testing at fourth grade was used as a basis for identifying low performing schools, I decided to conduct a study of fourth graders' problem solving strategies. In this study, I used task-based interviews to address the following question.

> When fourth grade students have had little formal experience with problem solving, what representations and strategies can they generate? In particular, how do they solve the tower problem and problems isomorphic to it?

Before beginning the interviews, I posed the tower problem to each of the three fourth grade classes in the school. The students worked in small groups to solve the problem. They were given cubes in two different colors. They also had paper, pencils, and crayons or markers to record their work. They were not told whether their answers were correct, but they were encouraged to justify their solutions. Class time was provided for students to share their ideas or to explain their work to the class. Although most groups found the correct answer, they struggled to explain their strategies. At the end of the class, each student was asked to write a letter to an absent student explaining the problem they had solved and their solutions. Many of the students incorporated drawings in their letters.

The tower problem had previously been published in *Measuring Up: Prototypes for Mathematics Assessment* (MSEB, 1993). I used the authors' suggested criteria to create the rubric exhibited in Figure 1. I used this to score the 60 classroom work samples. None of the work met the strictest criteria for "high" responses, but some work showed signs of strong mathematical reasoning. I selected 15 work samples at varying response levels for interview. The parents of nine of these students consented for their children to be interviewed. An additional five students did not participate in the classroom problem solving, and I gained consent to interview them. This made a total of 14 students.

Interviews followed the protocol outlined in Figure 2. In the interviews, students were asked to recall the problem (if they had solved it in the classroom), to extend it (building taller or shorter towers), and to record their work. They were again provided with cubes in two colors and with paper and markers.

Students were given time to solve the problem in their own way without interruption. When they were ready to talk about the problem, they were asked to justify their solutions. This often caused them to reorganize their physical towers or to change their recording methods. Interviews were audiotaped, field notes were taken, and digital pictures were taken after students organized their work. Any drawings or other written records were collected.

In the second interview, students solved the pizza problem, and these interviews were also audiotaped. The protocol for this interview is outlined in Figure 3. In both cases, the interview was extended to encourage students to verify their guesses or to seek recursive or generalized patterns for their solutions to the problem. A recursive pattern uses information from one step to find the solution for the next step. For example, students might notice that they could double the number of towers two cubes tall to predict the number of towers three cubes tall. A generalized pattern provides a rule or equation to describe the pattern. In this case, a general rule would be to find the number of possible towers using the expression 2^n, where *n* represents the number of cubes in the tower. For the pizza problem, students recorded their result on paper, usually either through drawings or through some sort of listing such as using abbreviations.

Based on the audio recordings and field notes, I scored each student's tower and pizza interviews using the interview analysis form exhibited in Figure 4. This gave me a total interview score for each student. Points were awarded for fluency, flexibility, and originality, as described in the table, providing a way to acknowledge more varied or more sophisticated approaches to the problems.

4. Results

4.1 Tower Problem

I presented the tower problem to the fourth grade classes early in the academic year. I was not able to interview the individual students until the winter, so several months passed since the classroom experience with the towers. For those who were engaged in the classroom problem solving, I reminded them of that activity. Most students remembered the activity but did not remember the solution. The primary advantage of having done the classroom activity seems to have been understanding that some organization was required. The experienced students immediately began to organize their work, while the others were less organized in their approaches. Each student was given cubes, markers, colored pencils, and a large piece of paper taped to the table. They were told that they could write on the paper if they liked. Most students started out using the paper as a mat to hold their cubes, but most eventually recorded something on the paper, either drawings or lists. A digital recording was made of any concrete manipulatives or records made by the students.

Several students began by making a tower then turning it upside down and building the tower with that color pattern. For example, a student might build a tower with a red cube on the bottom and two blue cubes on top. After turning the tower over, he might build a new tower with two blue cubes on the bottom and a red cube on top. The students called this the "flip" method. The problem with this method is that there are some towers that look the same when flipped. *Red, red, red* looks the same. *Blue, blue, blue*; *red blue red*; and *blue, red, blue* also look the same when flipped.

Another method used by students was the "opposite" method. A student might build a *red, blue, red* tower. Then the opposite tower would be *blue, red, blue*. Some students who started out using the flipping method also tried using opposites or a combination of flipping and opposites. The problem with both of these methods is that there is no logical way to determine when one can stop making towers and finding their opposites or flips. Most students eventually determined that there were eight possible towers, but they were often unable to justify their reasoning. They usually just said, "I can't think of any more."

Polya (1957) suggests that it is sometimes helpful to look at simpler cases when solving problems. I asked each student to tell me how many different towers one cube tall they could make. All immediately answered that there were two possible towers. Then I asked how many different towers could be made two cubes tall. Now students were more confident in explaining their answers. Several noted that a red tower one cube tall could generate two new towers by either being topped with blue or red. Similarly, a blue tower one cube tall could be topped with blue or red, yielding four towers two cubes tall.

As I talked with each student, I recorded their solutions in a table on the corner of the paper, whether or not the solutions were correct – one cube tall, two towers; two cubes tall, four towers; etc.

I asked each student to try to predict how many different towers could be made that were four cubes tall. Predictions ranged from 14 to 20, mostly based on guesses, but all students asserted that the answer would be an even number.

Actually building the towers four cubes tall was problematic. Finding flips and opposites was much more complicated, and students found themselves making duplicates by accident. When asked to record their work, most simply drew the towers or made marks to represent them. The manipulatives now seemed to interfere with finding more efficient solutions. Most students eventually found the correct answer of 16 towers, which I recorded in my table.

Next I asked each student to predict the number of different towers five cubes tall. Some students looked at the table and noticed that the number of different towers doubled each time the size of the tower grew by one cube, and they correctly predicted 32 towers. Others noticed the doubling but did not use this idea to make their predictions. Still others failed to notice the doubling pattern and predicted randomly.

4.2 Pizza Problem

In the second interview with each student, I posed the pizza problem with three available toppings – peppers, pepperoni, and sausage. I reminded the student that all pizzas already had cheese on them. This time there were no manipulatives available, but pencils, paper, and colored markers were available. Most students chose to draw the pizzas. Some used color coding or icons to represent the different toppings, but the students soon found that it was hard to keep track of the pizzas. Several students wrote the toppings above each pizza, and one decided to make a list of the pizzas instead of drawing. Another listed the toppings at the top of the page and circles at the bottom. Then she drew arrows from the toppings to the circles below to represent the different pizzas. However, this was very messy and the student was unable to generate all possible pizzas.

One student drew a Venn diagram with three intersecting circles, one for each topping (See Figure 5). Each area of the diagram represented a different pizza. For example, region I represents a pizza with pepperoni and peppers. Region II represents a pizza with all three toppings. The student counted seven pizzas. However, he could not account for the pizza with no toppings. After thinking for a while, he realized that the region outside the circles represented that pizza. This method was quite efficient for three available toppings, but it could get very complicated if the number of available toppings increased.

When I asked students to explain how they knew they had all possible pizzas, there was still some confusion, but there was much more evidence of organization. Students often started with one topping, then two toppings, then three. Several students originally forgot to consider the pizza with no toppings, but all eventually found all eight possible pizzas.

Next I asked each student to imagine what would happen if they had a small pizza stand and ran out of sausage. Now they only had two toppings available. I asked them to find how many different pizzas could now be made. Most students simply marked off all of the pizzas that had sausage on them. They found four different pizzas could be made. I then asked the students how many pizzas could be made if they ran out of pepperoni. Again, they marked off all pizzas with pepperoni and decided that only two pizzas could be made, pizza with no topping and pizza with peppers. As with the tower problem, I recorded the information in a table.

Next I asked the students to consider the three original toppings and to add mushrooms, making four available toppings. I asked them to predict how many different pizzas could be made, and there was wide variation in the predictions. I then asked each student to solve the problem. All students decided to add to their original solutions with three toppings. They realized that all of these pizzas could still be made.

Students generated new pizzas in two ways. Some students just started adding new pizzas to their original lists or drawings. This random method sometimes caused a pizza to be omitted. Most students seemed to realize that there should be an even number of pizzas, and they were able to find the missing ones. Other students methodically added mushrooms to each of their original eight pizzas, making a total of 16 different possible pizzas.

I added the results for four toppings to my table. For those students with whom I still had some available time, I asked them to predict the number of possible pizzas with five toppings available. Some actually wanted to draw or list all pizzas. Others recognized that the number of possible pizzas doubled, and they predicted 32 pizzas.

Some mentioned the doubling pattern but did not have confidence in their observations and guessed that 26 or 31 pizzas would be possible. My conversations with the students indicated that they were beginning to notice the structure of the problem and the underlying pattern. Some students mentioned their work with the tower problem, but it was not clear that they actually connected the structural similarities of the problems. However, all students exhibited growth in their ability to explain their work and justify their methods.

4.3 Analysis of Problem Solving

Using the interview analysis form (Figure 4), I assigned a score for each student's problem solving strategies. The resulting scores, along with demographics for each student, are recorded in Table 1. The median scores for black, white, female, and male students were not substantially different. The median score for students who participated in the classroom problem solving (20.5) was not higher than for those who did not (21).

This extended problem solving was new to the students interviewed in this study. Their representations were limited and their solution strategies were not always as sophisticated as those observed in Maher's study (Griswold, 2000). However, all students were interested in solving the problems and they persisted in working on the problems when they encountered obstacles or they were questioned about their work. Although they often lacked confidence in their abilities, some students were able to begin to recognize patterns and explain their strategies, the beginnings of proof.

5. Discussion

> This study centered on the following question.
>
> When fourth grade students have had little formal experience with problem solving, what representations and strategies can they generate? In particular, how do they solve the tower problem and problems isomorphic to it?

The children interviewed in this study struggled with many academic issues. They had widely varying success in mathematics. Their mathematics classes were primarily based on skills and applying algorithms. For this reason, their experience with representations such as tables, graphs, and even concrete manipulatives was very limited. However, once they became interested in solving a problem, they were able to draw on their natural problem solving skills and generate plausible solutions. Before starting a problem, each student had to consider how to begin and how to represent her work. This was especially true with the pizza problem, where students considered whether to draw pictures or make lists. They had to consider how to represent the toppings, perhaps with icons, with abbreviations, or with color coding. Once problems became more complex, students had to revise their thinking about how to extend their solutions or change their strategies.

As students worked through the problems, their comfort with problem solving clearly improved. I considered the job of the interviewer to be eliciting from each student as much information about their thinking as possible. This included the ability to self-correct. Therefore, through questioning, I gave students opportunities to reconsider or confirm their solutions.

Throughout the study, the students moved from random approaches to the tower problem to organizing their strategies as well as their solutions. Their representations generally became more sophisticated and their strategies became more efficient. In general, the students were able to solve the pizza problem more quickly than the tower problem with which they began. Some students began to recognize the importance of structure and pattern recognition.

Based on the interviews and my observations of the work of each student, the major reason for these changes seems to have been the opportunity to engage in solving rich mathematical problems, to think about their own thinking and to try to explain it to others. A reasonable extension might be to provide more such problem solving experiences for these students. This, of course, would require that teachers have the necessary expertise to select and pose rich mathematics problems.

Although this study is limited in scope and cannot be generalized to a larger population, the results may inform the work of other researchers in the field of problem solving, especially research with students who have little experience with problem solving. Future studies might address introduction of problem solving in mathematics classrooms or interviews to determine the problem solving strategies of teachers themselves.

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Response Level	Characteristics
High	The high response shows recognition of the need for a systematic scheme to keep track of "all possibilities" in a way that supports a conclusion that there could not be any other towers of height three. The student reasoning does not rely on the argument that "I cannot think of any others," but instead presents some reasonable scheme that is potentially exhaustive. (MSEB, 1993, p. 138) Examples: Proof by Cases - There is only one tower with no blues, there are three towers, with exactly one blue, etc. Proof by Induction - Using the number of towers two cubes tall to discover the number of towers three cubes tall by adding cubes to those.
Medium	The response shows some suggestion of a method for being exhaustive, but shows no recognition that this feature is present or that it is needed. There may also be explicit statements to the effect that "I couldn't find any more." An answer qualifies as medium if it presents a proof of some important part of the problem -- for example, that the number of towers must be even because every tower has exactly one "opposite" by interchanging the colors. (MSEB, 1993, p. 139)
Low	The letter describes one or more methods for generating new towers, but fails to deal with the question of devising a method that will exhaustively produce all possible towers, and shows no recognition of the need for such a method. (MSEB, 1993, p. 139)

Figure 1: Scoring Guide for Classroom Artifacts for Tower Problem

Pose Question – Tower Problem
How many different towers three cubes high can be created by choosing from cubes of two colors?
A. Ask student to recall the classroom session and describe what happened.
B. Question student to clarify fact that towers of a single color can be created.
C. Make sure student understands that Red, Red, Blue is different from Blue, Red, Red.
D. Have student work problem.
Are you sure you have all possible towers? How do you know?
How could we make a record of your solution?
Suppose we decide to make towers four cubes high. How many different towers could we make?
If student attempts problem, how do you know you have all possible towers?
If student doesn't want to attempt the problem, how many different towers could we make two cubes tall? How do you know if you have all possible towers?
What have we found out so far? Could we make a record of what we know? If student doesn't suggest something, simply write down what we know.
Could we do anything else? Could we make towers five cubes tall? Is there a way we could figure this out without actually making the towers?
Have you ever done problems like this before? (Probe to find out more about student's past experiences.)

Figure 2: Protocol for Tower Problem Interview

Pose Question – Pizza Problem
How many different pizzas can you make when choosing from three different toppings: peppers, pepperoni, and sausage? For the purposes of this problem, you cannot make half pizzas like half peppers and half sausage. However, you can make a pepper and sausage pizza.
A. Work with student to give one example of a possible pizza.
B. Make pencils, markers, and paper available for student to begin representing solutions to the problem. First limit the number of toppings to three. Then work with students to find the number of pizzas for two and one topping. Finally, ask student to guess how many pizzas can be made with four toppings and then make them.
C. Provide time for student to work problem. Pay attention to whether or not student makes a pizza with no ingredients. This would be a plain cheese pizza.
Are you sure you have all possible pizzas? How do you know?
How could we make a record of your solution?
Suppose we have five different ingredients. How many different pizzas do you think you could make?
If student has a guess, have him or her explain the guess.

Figure 3: Protocol for Pizza Problem Interview

Student Number _______			
Characteristic	**Characteristic Present?** (One point for each unique occurrence.)		**Notes** (Example: Was the response prompted or generated by student?)
	Towers	Pizza	
Correct Answer			
Level 1 - Concrete Methods			
Manipulatives			
Drawing			
Listing			
Level 2 - Systematic Approach			
No systematic approach (no points awarded)	----------	----------	
Clear system of enumeration			
Looking for opposites			
Flips			
Diagrams			
Other systematic approaches observed			
Level 3 – Systematic Approach Leading to Proof			
Fruitful methods – beginning of proof			
Clear justification for answer			
Proof by case			
Justifying doubling			
Making a Record (Communicating ideas when asked to make a record of the problem solving.)			
Drawing			
Using icons, letters or other notation			
Color coding			
Making a list			
Written description			
Making a grid or table			
Writing a recursive description			
Writing a recursive equation			
Writing a generalized description			
Writing a generalized equation			
Extensions			
Simpler case			
More complex case			
Pattern Recognition			
Recursive pattern			
Informal statement of pattern with some indication of pattern.			
Formal statement of pattern			
From list			
From table			
From graph			
Generalized pattern			
Informal statement of pattern with some indication of pattern.			
Formal statement of pattern			
From list			
From table			
From graph			
Fluency			
(How many different solutions?)			
Flexibility			
(How many different mathematical ideas are discovered in the above solutions?)			
Originality			
(How many responses represent a high quality of mathematical thinking?)			
Correctly predicts for more complex problems (May or may not have identified a pattern.)			
Total Number of Occurrences			
Total for Tower and Pizza Problems			

Figure 4: Interview Analysis Form

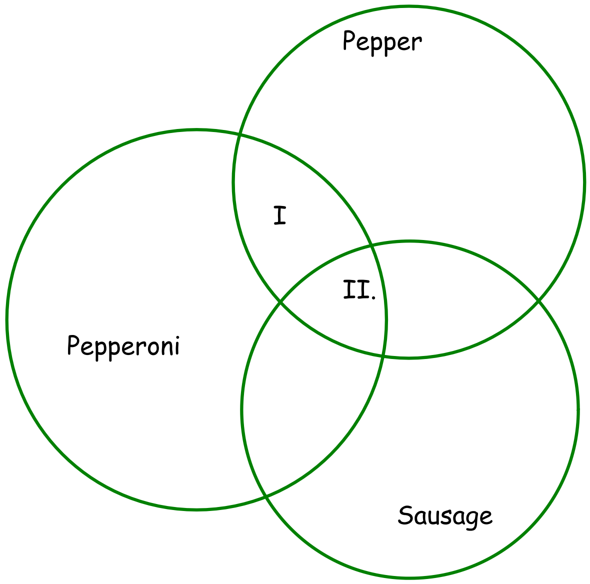

Figure 5: Venn Diagram for Pizzas with Three Available Toppings

Black	White	Female	Male	Participated in Classroom problem solving activity?	Score
X		X		X	29
X		X		X	25
	X		X		22.5
	X	X			22
X			X	X	22
X			X	X	22
X		X			21
	X	X		X	20.5
	X	X			20
X			X	X	20
	X		X	X	19
X			X	X	17
X		X			17
X		X		X	16
21	20.5	20.75	21	20.5	20.75
Median Scores					

Table 1: Scores for Tower Problem and Pizza Problem by Race, Gender, and Participation in Classroom Problem Solving (N=14).